$$\dot{\vec{x}} = A\vec{x} \quad (1)$$

EIGENVALUES $\lambda_1, \dots, \lambda_n$

$\mathrm{Re}(\lambda_i) < 0 \ \forall i$

$$\downarrow$$

$\vec{x}(t) \equiv \vec{0}$ ASYMP. STABLE

$$\downarrow$$

$V(\vec{x}) = \vec{x}^T K \vec{x}$ WHERE $K = \int_0^\infty e^{A^T t} e^{At}\, dt$ IS A STRONG LIAPUNOV FCT

$$\downarrow$$

$$\dot{\vec{x}} = A\vec{x} + R(\vec{x}) \quad (2)$$

$$R(\vec{0}) = \vec{0}$$

$$\lim_{\|\vec{x}\| \to 0} \frac{\|R(\vec{x})\|}{\|\vec{x}\|} = 0$$

$$\downarrow$$

$\exists \delta > 0$ ST $V : B_\delta(\vec{0}) \to \mathbb{R}$ IS A STRONG LIAPUNOV FCT FOR (1) <u>AND</u> (2)

$$\downarrow$$

$\vec{x}(t) \equiv \vec{0}$ ASYMP. STABLE

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$\lambda_i \neq \lambda_j$ FOR $i \neq j$, $\mathrm{Re}(\lambda_i) \neq 0 \ \forall i$.
$\mathrm{Re}(\lambda_i) > 0$ FOR SOME $i$

$$\downarrow$$

$\exists P$ INV AND

$$B = \begin{pmatrix} B_1 & & 0 \\ & \ddots & \\ 0 & & B_r \end{pmatrix}$$

$\left( B_i = \lambda_j \text{ OR } B_i = \begin{pmatrix} \mathrm{Re}(\lambda_j) & -\mathrm{Im}(\lambda_j) \\ \mathrm{Im}(\lambda_j) & \mathrm{Re}(\lambda_j) \end{pmatrix} \right)$

ST $A = PBP^{-1}$

$$\downarrow \ \vec{z} = P^{-1}\vec{x}$$

$$\dot{\vec{z}} = B\vec{z}$$

$$\downarrow \ \tilde{U}(\vec{z}) = \vec{z}^T B \vec{z}$$

$\vec{z}(t) \equiv \vec{0}$ IS UNSTABLE

$$\downarrow \ U(\vec{x}) = \vec{x}^T (P^{-1})^T B P^{-1} \vec{x}$$

$\vec{x}(t) \equiv \vec{0}$ IS UNSTABLE

$$\downarrow$$

$$\dot{\vec{x}} = A\vec{x} + R(\vec{x})$$

$$R(\vec{0}) = \vec{0}$$

$$\lim_{\|\vec{x}\| \to 0} \frac{\|R(\vec{x})\|}{\|\vec{x}\|} = 0$$

$$\downarrow \ U(\vec{x})$$

$\vec{x}(t) \equiv \vec{0}$ IS UNSTABLE.