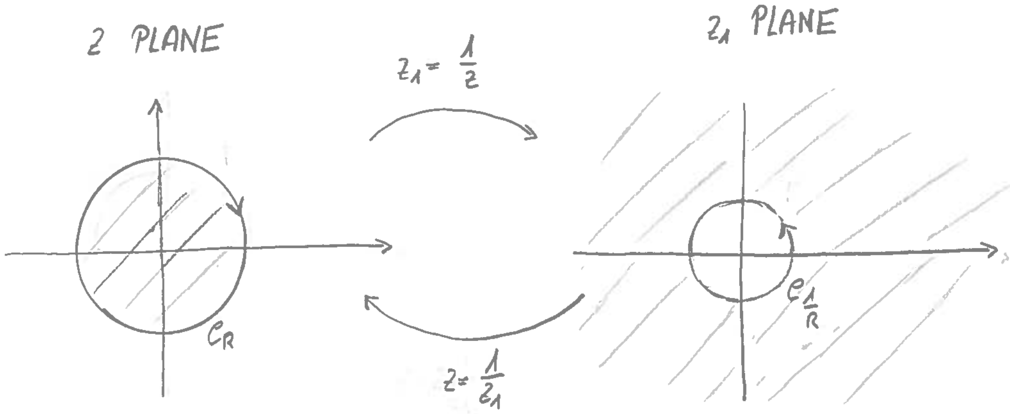
I_∞ INDEX AT ∞

$$\dot{x} = f(x,y)$$
$$\dot{y} = g(x,y)$$

$$\Big\Downarrow \; z = x + iy$$

$$\dot{z} = F(z) = f(x,y) + i\,g(x,y)$$
$$\phi(x,y) = Arg(F(z))$$

CHOOSE $R>0$ ST (x_0,y_0) EP $\Rightarrow \|(x_0,y_0)\|<R$



$$\Downarrow$$

$$I_\infty = 2 - I_{C_R}$$

$$\dot{\vec{x}} = f(\vec{x}, \vec{\mu})$$

$\vec{x}(t) \in \mathbb{R}^n$

$\mu \in \mathbb{R}^m$ ... PARAMETER

$$\vec{y}(t) = \begin{pmatrix} \vec{x}(t) \\ \vec{\mu} \end{pmatrix} \in \mathbb{R}^{n+m},$$

$$g : \mathbb{R}^{n+m} \to \mathbb{R}^{n+m}; \quad \vec{y} \longmapsto g(\vec{y}) = \begin{pmatrix} f(\vec{y}) \\ 0 \end{pmatrix}$$

$$\dot{\vec{y}} = g(\vec{y})$$

g ... LIPSCHITZ:

y_1, y_2 SOLUTIONS

$$|y_2 - y_1|' \leq L|y_2 - y_1|.$$

GRONWALL

$$|y_2(t) - y_1(t)| \leq e^{Lt} |y_2(0) - y_1(0)|.$$

SMALL CHANGES IN μ CAN LEAD TO LARGE CHANGES IN THE DYNAMICS

CHANGE IN THE NUMBER OF EQUILIBRIUM POINTS.

CHANGE IN THE STABILITY OF EQUILIBRIUM POINTS

μ_0 BIFURCATION POINT.

$\triangleq$ PARAMETER WHERE THE CHANGE OCCURS.