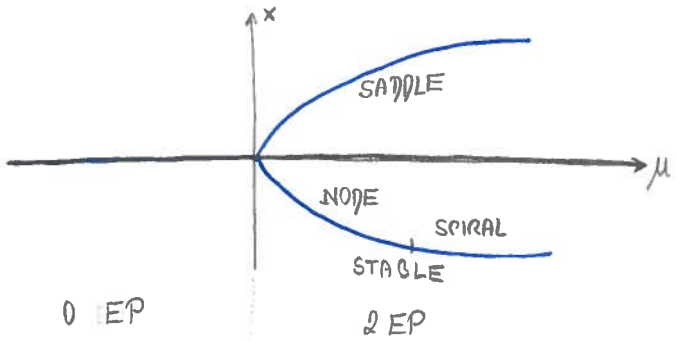
$$\dot{\vec{x}} = f(\vec{x}, \vec{\mu})$$

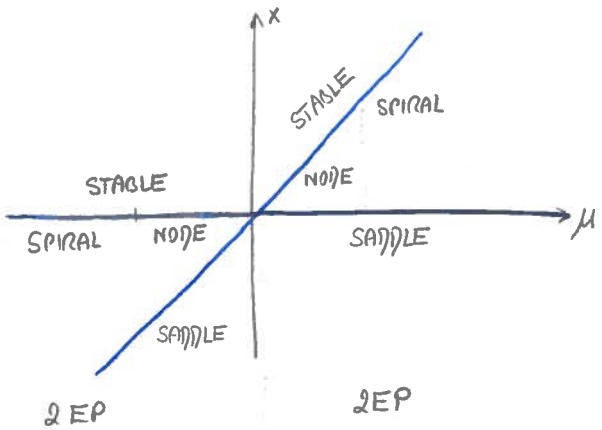
$\vec{x}(t) \in \mathbb{R}^n$

$\mu \in \mathbb{R}^m$ PARAMETER

• SADDLE-NODE BIFURCATION

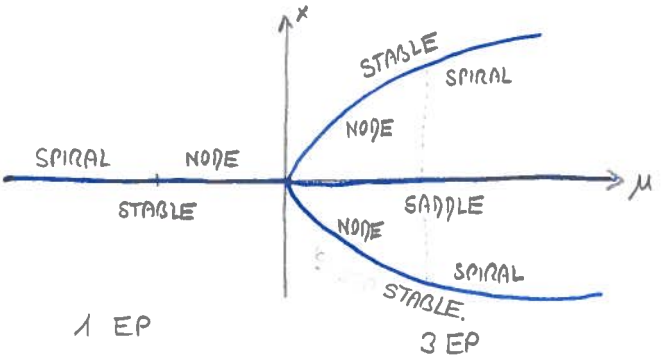



• TRANSCRITICAL BIFURCATION




EXCHANGE STABILITY UPON COLLISION

• PITCHFORK BIFURCATION



$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} \mu & 1 \\ -1 & \mu \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} - \beta(r)\begin{pmatrix} x \\ y \end{pmatrix}$$

$r = \sqrt{x^2+y^2}$

$\beta(0)=0,\ \beta(r)>0\ r>0$

$\beta(r)$ CONT, STRICTLY INCREASING

$\beta(r) = \Theta(r)$ ($r$ CLOSE TO 0)

↓ POLAR COORDINATES

EP: $(0,0)$

$\mu>0$ UNSTABLE SPIRAL

$\mu<0$ STABLE SPIRAL

$\mu=0$ ?

↓ POLAR COORDINATES

$(r^2)' = 2r^2(\mu-\beta(r))$

$\theta' = -1$

$\mu \leq 0$ → STABLE SPIRAL

$\mu>0$ → $\exists!$ PERIODIC SOLUTION

$\hat{=}$ CIRCLE WITH RADIUS $\beta^{-1}(\mu)$

↓

STABLE LIMIT CYCLE