$\dot{\vec{x}} = f(\vec{x})$

$f: \mathcal{U} \subseteq \mathbb{R}^2 \to \mathbb{R}^2$ LIP. CONT

$\mathcal{U} \subseteq \mathbb{R}^2$ OPEN.

$\downarrow$

$\vec{x}(t)$ SOLUTION ST

- $\vec{x}(0) = \vec{x}_0$
- $\exists$ K COMPACT ST $\mathcal{U}_{\vec{x}_0} \subseteq K \subseteq \mathcal{U}$

$\downarrow$ $\omega(\mathcal{U}_{\vec{x}_0}) = \{\vec{z} \in \mathbb{R}^2 \mid \exists t_n \to \infty \text{ ST: } \lim \vec{x}(t_n) = \vec{z}\}$

$\omega(\mathcal{U}_{\vec{x}_0})$ NON-EMPTY
COMPACT
CONNECTED

$\downarrow$ $y_0 \in \omega(\mathcal{U}_{\vec{x}_0})$ AND $y(t)$ CORRESP SOLUTION

$y(t) \in \omega(\mathcal{U}_{\vec{x}_0})$ $\forall t$

$\downarrow$ $E = \{\vec{z} \in \mathbb{R}^2 \mid \vec{z} \text{ EQUILIBRIUM POINT}\}$.

$E \cap \omega(\mathcal{U}_{\vec{x}_0}) = \phi$ $\qquad$ $E \cap \omega(\mathcal{U}_{\vec{x}_0}) \neq \phi$

$\downarrow$ $\qquad$ $\downarrow$

$\omega(\mathcal{U}_{\vec{x}_0}) =$ CLOSED PHASE PATH
$\triangleq$ PERIODIC SOLUTION

$\omega(\mathcal{U}_{\vec{x}_0}) = \begin{cases} \text{EQUILIBRIUM POINT} \\ \text{EQUILIBRIUM POINT + PHASE PATHS JOINING THEM} \end{cases}$