$$\dot{\vec{x}}(t) = f(\vec{x}(t))$$
$$\vec{x}(0) = \vec{a}$$
$$(\vec{a} \in \mathbb{R}^n,\ f: \Omega \to \mathbb{R}^n,\ a \in \Omega \subseteq \mathbb{R}^n)$$

$\Downarrow$ f LOCALLY LIPSCHITZ CONTINUOUS

UNIQUE LOCAL SOLUTION

$$\vec{x}: [-T, T] \to \mathbb{R}^n$$

$\exists \lim_{t \to T^-} \vec{x}(t) \in \Omega$

$\vec{x}$ CAN BE EXTENDED

$$\vec{x}: [-T, T+T_1] \to \mathbb{R}^n$$

$\lim_{t \to T^-} \vec{x}(t) \notin \Omega$ OR $\lim_{t \to T^-} \vec{x}(t) = \pm\infty$

$\vec{x}$ CAN'T BE EXTENDED IN THE POSITIVE DIRECTION

$\Downarrow$

MAXIMAL INTERVAL OF EXISTENCE

$$\vec{x}: (\alpha, \beta) \to \mathbb{R}^n.$$

$\alpha = -\infty$, $\beta = \infty$

GLOBAL SOLUTION

$\alpha > -\infty$

$\lim_{t \to \alpha^+} x(t) = \pm\infty$

OR

$\lim_{t \to \alpha^+} x(t) \in \partial\Omega$

$\beta < \infty$

$\lim_{t \to \beta^-} x(t) = \pm\infty$

OR

$\lim_{t \to \beta^-} x(t) \in \partial\Omega.$

OBS: $\dot{\vec{x}} = f(\vec{x}(t), t)$, $\vec{x}(0) = \vec{a}$ $\Longrightarrow$ $\dot{\vec{y}}(t) = g(\vec{y}(t))$, $\vec{y}(0) = \vec{b}$

$$\vec{y}(t) = \begin{pmatrix} \vec{x}(t) \\ t \end{pmatrix}$$
$$\vec{y}(0) = \begin{pmatrix} \vec{a} \\ 0 \end{pmatrix}$$