$$\dot{\vec{x}} = A\vec{x}$$

$(\vec{x}: I \subseteq \mathbb{R} \to \mathbb{R}^2$, $A$ $2\times 2$ MATRIX$)$

$\downarrow$

PHASE DIAGRAM DEPENDS ON THE EIGENVALUES $\lambda_1$ AND $\lambda_2$ OF $A$ AND THE CORRESPONDING EIGENVECTORS.

- $\lambda_1 \neq \lambda_2 \in \mathbb{R}$
- $\lambda_1 = \lambda_2$. TWO LIN. IND EIGENVECTORS
- $\lambda_1 = \lambda_2$. ONE LIN. IND EIGENVECTOR.
- $\lambda_1 = \bar{\lambda}_2 \in \mathbb{C} \setminus \{\mathbb{R}\}$.

IDEA: $A, B$ ... $2\times 2$ MATRICES, $P$ ... INV $2\times 2$ MATRIX $\Big\}$ SUCH THAT $A = PBP^{-1}$

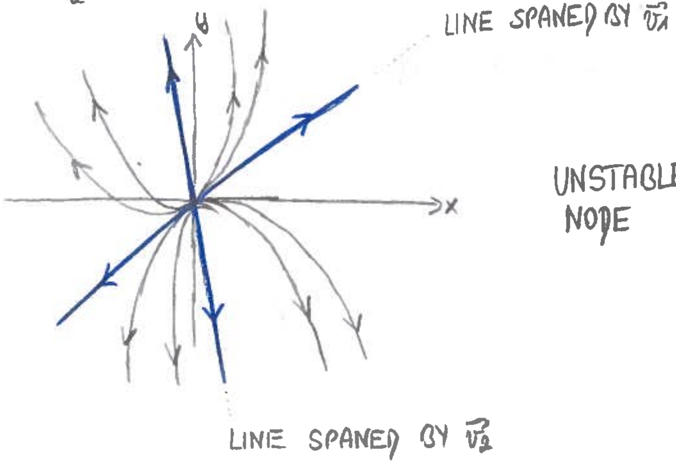
$\Downarrow$

$$\begin{tikzcd}
\dot{\vec{x}} = A\vec{x} \arrow[rr, bend left, "\vec{z} = P^{-1}\vec{x}"] & & \dot{\vec{z}} = B\vec{z} \arrow[ll, bend left, "\vec{x} = P\vec{z}"]
\end{tikzcd}$$

$\dot{\vec{x}} = A\vec{x}$ WITH $\det(A) \neq 0$

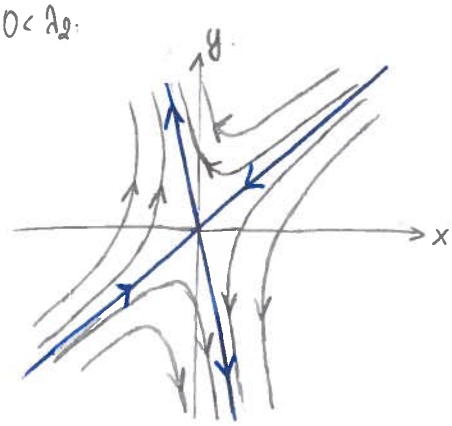
(i) $A$ ... $\lambda_1 \neq \lambda_2 \in \mathbb{R}$, $A\vec{v_1} = \lambda_1 \vec{v_1}$, $A\vec{v_2} = \lambda_2 \vec{v_2}$

·) $0 < \lambda_1 < \lambda_2$.



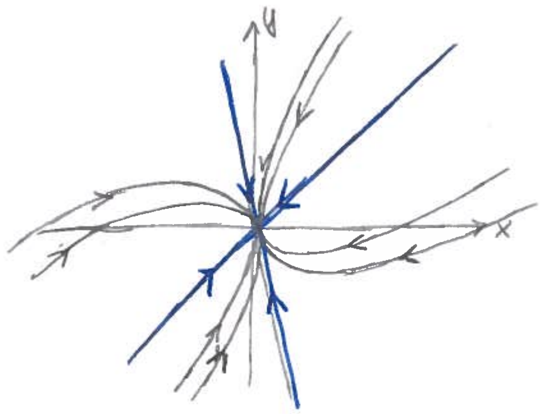

UNSTABLE NODE

·) $\lambda_1 < 0 < \lambda_2$.




SADDLEPOINT

·) $\lambda_1 < \lambda_2 < 0$.




STABLE NODE

SOLUTION IS OF THE FORM

$$\vec{x}(t) = Be^{\lambda_1 t}\vec{v_1} + Ce^{\lambda_2 t}\vec{v_2} \quad (B, C \in \mathbb{R})$$