

$$\dot{x} = f(x, y)$$

$$\dot{y} = g(x, y)$$

$$\Downarrow (0,0) \text{ EQUILIBRIUM POINT}$$

$$\begin{aligned} f(x, y) &= f(0,0) + f_x(0,0)x + f_y(0,0)y + F(x, y) \\ &= 0 + ax + by + F(x, y). \end{aligned}$$

$$\text{WHERE } (x, y) \approx (0,0) \text{ AND } F(x, y) = \mathcal{O}(\|(x, y)\|^2)$$

$$g(x, y) = cx + dy + G(x, y)$$

$$\text{WHERE } (x, y) \approx (0,0) \quad G(x, y) = \mathcal{O}(\|(x, y)\|^2).$$

$$\Downarrow \vec{x} = (x, y).$$

$$\dot{\vec{x}} = A\vec{x} + H(\vec{x})$$

$$\left(A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad H(\vec{x}) = \begin{pmatrix} F(x, y) \\ G(x, y) \end{pmatrix} = \mathcal{O}(\|\vec{x}\|^2) \right)$$

$$\Downarrow \begin{aligned} A &= PCP^{-1} \\ P &\text{ INVERTIBLE.} \end{aligned}$$

$$\dot{\vec{x}} = PCP^{-1}\vec{x} + H(PCP^{-1}\vec{x}).$$

$$\Downarrow \vec{z} = P^{-1}\vec{x}.$$

$$\dot{\vec{z}} = C\vec{z} + \underbrace{P^{-1}H(P\vec{z})}_{\mathcal{O}(\|\vec{z}\|^2)}.$$