$$\dot{\vec{x}} = F(\vec{x})$$

$$(\vec{x}: I \subseteq \mathbb{R} \to \mathbb{R}^2,\ F: U \subseteq \mathbb{R}^2 \to \mathbb{R}^2)$$

$$\Big\Downarrow \ \vec{0} \text{ EQUILIBRIUMSPOINT}$$

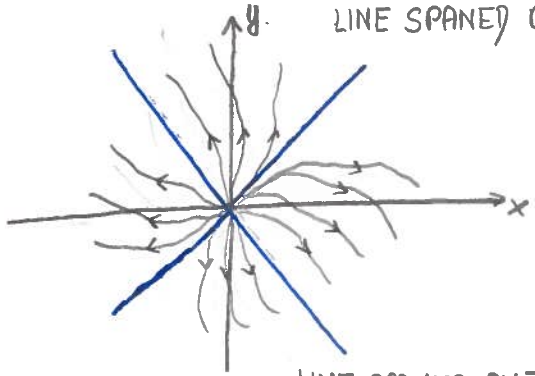
$$\dot{\vec{x}} = A\vec{x} + G(\vec{x})$$

$$(A\ 2\times 2 \text{ MATRIX},\ G(\vec{x}) = O(|\vec{x}|^2) \text{ FOR } \vec{x} \approx \vec{0})$$

LOCAL BEHAVIOR.

- A TWO REAL EIGENVALUES $0 < \lambda_1 < \lambda_2$, $A\vec{v_1} = \lambda_1 \vec{v_1}$ AND $A\vec{v_2} = \lambda_2 \vec{v_2}$

LINE SPANED BY $\vec{v_1}$

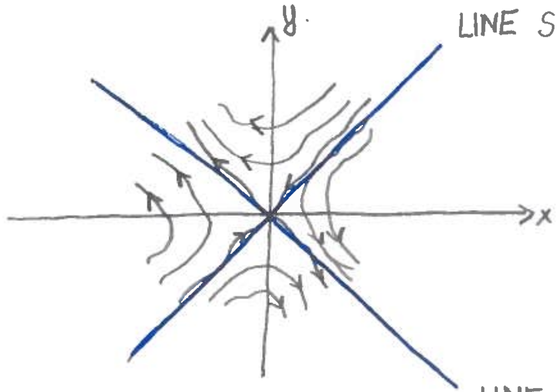


LINE SPANED BY $\vec{v_2}$

(REVERSED ARROWS FOR $\lambda_2 < \lambda_1 < 0$!)

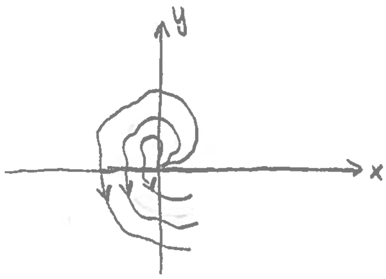
- A TWO REAL EIGENVALUES $\lambda_1 < 0 < \lambda_2$, $A\vec{v_1} = \lambda_1 \vec{v_1}$ AND $A\vec{v_2} = \lambda_2 \vec{v_2}$

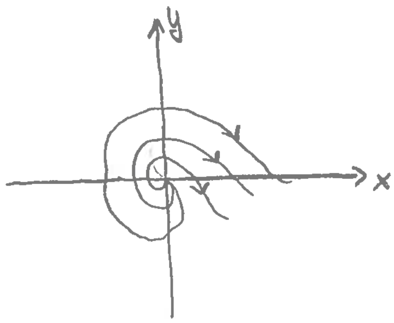
LINE SPANED BY $\vec{v_1}$



LINE SPANED BY $\vec{v_2}$

- A TWO COMPLEX EIGENVALUES $\lambda_1 = \overline{\lambda_2}$, $\mathrm{Re}(\lambda_1) > 0$





$$\dot{x} = f(x,y)$$
$$\dot{y} = g(x,y)$$

WITH EQUILIBRIUMPOINT (x_0,y_0)

$$\Downarrow$$

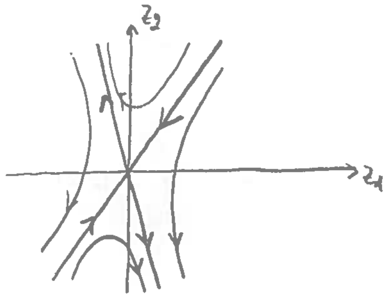
BEHAVIOR NEAR (x_0,y_0) CAN BE READ OFF FROM

$$\dot{\vec{z}} = A\vec{z} \quad \text{WHERE} \quad A = \begin{pmatrix} f_x(x_0,y_0) & f_y(x_0,y_0) \\ g_x(x_0,y_0) & g_y(x_0,y_0) \end{pmatrix}$$

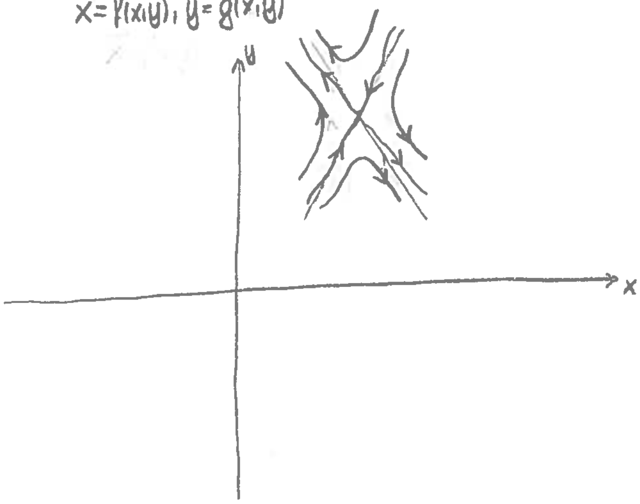
(EIGENVALUES OF A MUST SATISFY $\mathrm{Re} \neq 0$)

IDEA

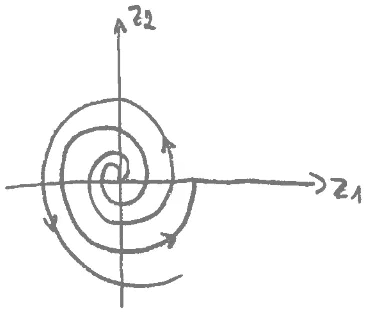
$\dot{\vec{z}} = A\vec{z}$



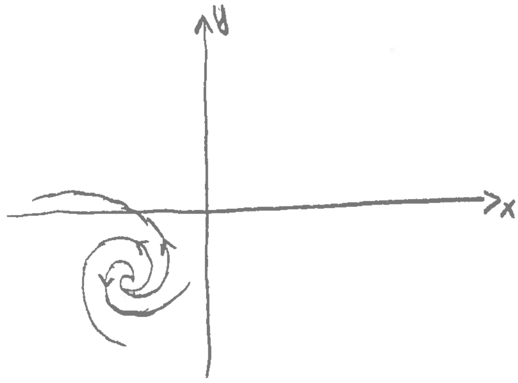
$\dot{x} = f(x,y)$, $\dot{y} = g(x,y)$



$\dot{\vec{z}} = A\vec{z}$



$\dot{x} = f(x,y)$, $\dot{y} = g(x,y)$



REMAINING PARTS FROM THE ARROWS!