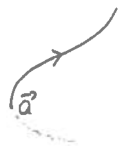
$\dot{\vec{x}}(t) = f(\vec{x}(t))$

UNIQUE SOLUTIONS

$\vec{x}^*(t)$ SOLUTION WITH $\vec{x}^*(t_0) = \vec{a}$

$\Downarrow$



$\mathcal{H}_a$

$+$ = PHASE PATH

$\mathcal{H}_a = \{ z \in \mathbb{R}^n : \exists t \geq t_0 \text{ ST } \vec{x}^*(t) = z \}.$

$\Downarrow$ $\forall \varepsilon > 0 \ \exists \delta > 0$ ST $\|a - b\| < \delta \Rightarrow \max\limits_{z \in \mathcal{H}_b} \text{dist}(z, \mathcal{H}_a) < \varepsilon$

$\mathcal{H}_a$ POINCARÉ STABLE

$\mathcal{H}_a$.

$\mathcal{H}_b$.

$\Downarrow$ $\forall \varepsilon > 0 \ \exists \delta > 0$ ST $\|\vec{x}^*(t_0) - \vec{x}(t_0)\| < \delta \Rightarrow \|\vec{x}(t) - \vec{x}^*(t)\| < \varepsilon \ \forall t \geq t_0$.

$\vec{x}^*(t)$ LIAPUNOV STABLE FOR $t \geq t_0$

$\|$

UNIFORMLY STABLE FOR $t \geq t_0$

$\Downarrow$ $\exists \eta$ ST $\|\vec{x}^*(t_0) - \vec{x}(t_0)\| < \eta \Rightarrow \lim\limits_{t \to \infty} \|\vec{x}^*(t) - \vec{x}(t)\| = 0$

$\vec{x}^*(t)$ ASYMPTOTICALLY STABLE FOR $t \geq t_0$