

Homework 03

Exercises from the textbook:

8.1. Consider two consecutive discrete Fourier transforms:

$$\mathcal{F}_N : (y_k) \mapsto (Y_n) \text{ and } \mathcal{F}_n : (Y_n) \mapsto (z_l).$$

Compute z_l as a function of y_k .

8.2. Let (x_k) and (y_k) be two complex periodic sequences with period N such that

$$x_{N-k} = \bar{x}_k \text{ and } y_{N-k} = \bar{y}_k$$

for all integer k . Show that their discrete Fourier transforms (X_n) and (Y_n) are real and that they can be computed with a single transform of order N .

8.3. Compute the successive powers of the matrix Ω_N .

8.5. Calculate the discrete Fourier transform of the vector $x_k = k$, $k = 0, 1, 2, \dots, N-1$

Exercise about Hermite functions:

Let

$$h_n(t) = e^{t^2/2} \frac{d^n}{dt^n} e^{-t^2}, \quad n = 0, 1, 2, \dots$$

Calculate the L^2 norm:

$$\int_{-\infty}^{\infty} h_n^2(t) dt.$$