Lecture 02, 09.01.2014

- Reminding: Fourier series, formulas for the coefficients
- The space $L^1(-\pi, \pi)$. A function should be in L^1 in order to the expression for $c_n(f)$ makes sense.
- Reminding: Cauchy-Schwartz inequalities for integrals and series.
- $L^2(-\pi, \pi) \subset L^1(-\pi, \pi)$, inverse is not true, example.
- Formulation of the Riemann-Lebesgue lemma.
- Idea of the proof
- Step function and its Fourier coefficients. *Exercise:* the Fourier coefficients tends to zero as $n \to \pm\infty$.
- Approximation fact
- Proof of the Riemann-Lebesgue lemma
- Digression:
  - Spectra of a signal, frequencies
  - Examples
  - *Exercise:* evaluate the frequency of the radar radiation given resolution
  - further harmonic models: heart rhythms, brain rhythms, water waves
- Decay of the Fourier coefficients of periodic functions with derivatives in $L^2(-\pi, \pi)$
- *Exercise:* describe infinitely smooth periodic functions in terms of the decay of their Fourier coefficients
- Absolute and uniform convergence of the Fourier series for periodic functins with derivatives in $L^2(-\pi, \pi)$.
- Setting of the question of the local convergence
- Expression of the partial sum of the Fourier series in terms of the Dirichlet kernel.
- Explicit formula for the Dirichlet kernel