FOURIER ANALYSIS

PROBLEM LIST 4

Problem 1. Show that $e^{-|x|}$ is not differentiable on $\mathbb{R}$.

Problem 2. Show that e^{-x^2} is an infinitely differentiable function on $\mathbb{R}$ and that $\lim_{|x|\to\infty} \frac{d^n}{dx^n} e^{-x^2} = 0$ for every $n \in \mathbb{N}$.

Problem 3. Show that $(1+|x|^m)\frac{d^n}{dx^n}e^{-x^2}$ is bounded for every $m, n \in \mathbb{N}$.

Problem 4. Show that $f(x) = e^{-x^{-2}}$ for $x \neq 0$, $f(0) = 0$ is an infinitely differentiable function and that $\frac{d^n}{dx^n} f(0) = 0$ for every $n \in \mathbb{N}$.

Problem 5. Construct a function which is infinitely differentiable and compactly supported. Hint: use f from the previous problem.

Problem 6 (Heat kernel, important!)**.** Let

$$h_t(x) = \frac{1}{\sqrt{t}} \exp\left(\frac{-\pi x^2}{t}\right)$$

Show that $\int_{\mathbb{R}} h_t(x)\,dx = 1$ and that for every $\delta > 0$

$$\lim_{t\to 0} \int_{|x|>\delta} h_t(x)\,dx = 0.$$