- $f_n \in \mathcal{O}(\Omega)$, $\Omega \subset \mathbb{C}$ open set

- $f_n \longrightarrow f$ uniformly in compacts of Ω

$\Leftrightarrow$ For any $K \subset \Omega$ we have $f_n \longrightarrow f$ uniformly in K

$\Leftrightarrow$ For any $K \subset \Omega$: $\forall \varepsilon > 0$ $\exists n_0 \in \mathbb{N}$: $n \geq n_0$: $|f_n(z) - f(z)| < \varepsilon$ $\forall z \in K$

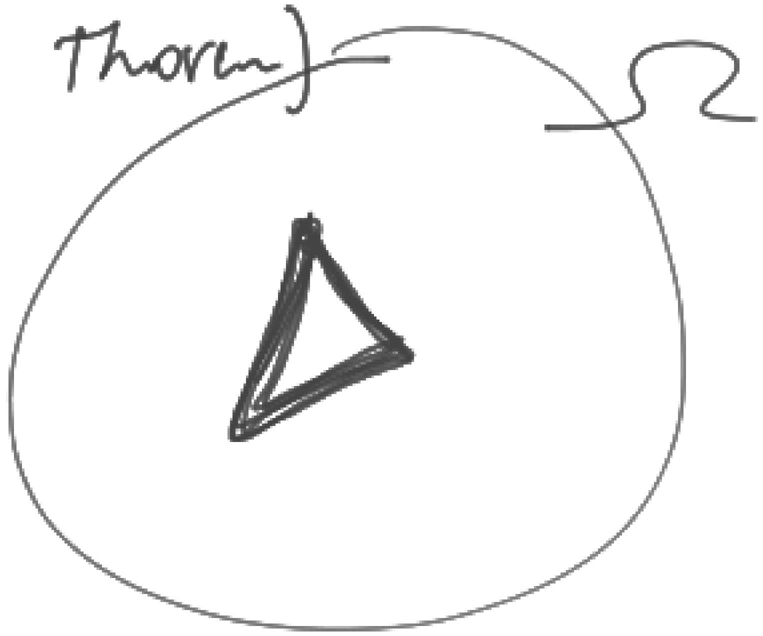
$\downarrow$

$n_0(\varepsilon, K)$

$\Rightarrow f \in \mathcal{O}(\Omega)$

- $f : \Omega \to \mathbb{C}$, $\Omega \subset \mathbb{C}$ open set
- f is continuous on Ω
- $\int_{\partial\Delta} f(z)\, dz = 0$ $\forall \Delta \subset \Omega$,

$\Rightarrow$ (Morera's Theorem) $\Rightarrow f \in \mathcal{O}(\Omega)$



EXAMPLE

$g : [-1,1] \to \mathbb{C}$ continuous

Define:

$$H(z) = \int_{-1}^{1} g(t) e^{izt}\, dt$$

i) $H : \mathbb{C} \to \mathbb{C}$

ii) $|H(z+h) - H(z)| \leq \int_{-1}^{1} |g(t)| |e^{izt}| |e^{iht} - 1|\, dt$

$|t| \leq 1$

$|h| \leq 1$

$\ll |ht|$

iii) $$\int_{\partial\Delta} H(z)\,dz = \int_{\partial\Delta} \left(\int_{-1}^{1} g(t) e^{izt}\, dt \right) dz$$

$$= \int_{-1}^{1} g(t) \left(\underbrace{\int_{\partial\Delta} e^{izt}\, dz}_{} \right) dt$$

$$\int_{-1}^{1} g(t)\, 0\, dt = 0$$

$f \in \mathcal{O}(\Omega)$

$$\Rightarrow \int_{\partial\Delta} f(z)\,dz = 0$$

$\Delta \subset \Omega$

e^{izt}

$\Rightarrow H \in \mathcal{O}(\mathbb{C})$

T- WEIERSTRASS

$f_n : \Omega \to \mathbb{C}$ functions

i) $|f_n(z)| \leq a_n \quad \forall n \in \mathbb{N}$

ii) $\sum_{n \in \mathbb{N}} a_n < \infty$

$$\Longrightarrow \quad \sum_{n=1}^{N} f_n(z) \longrightarrow \sum_{n=1}^{\infty} f_n(z)$$

absolutely and uniformly in Ω

K

$f_n \in \mathcal{O}(\Omega)$, K $\quad \Longrightarrow \quad \sum_{n \in \mathbb{N}} f_n(z)$ analytic function

PROBLEM 69:

$f \in M(\mathbb{R})$, $\hat{f} \in M(\mathbb{R}) \Rightarrow \sum_{k\in\mathbb{Z}} \hat{f}(k) = \sum_{n\in\mathbb{Z}} f(n)$ (P.S.F with $x=0$)

$\sum \hat{f}(k) e^{2\pi i k x} = \sum f(x+n)$

$\operatorname{supp} \hat{f} \subset [-1,1]$

$f(n) \geq e^{-2\pi|n|}$

$$\hat{f}(0) = \sum_{n\in\mathbb{Z}} f(n) \geq \sum_{n\in\mathbb{Z}} e^{-2\pi|n|}$$

$$\frac{e^{2\pi}+1}{e^{2\pi}-1}$$