

WRITING THE PROJECT REPORT

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ABSTRACT. The project reports in Optimisation I are expected to be written in the form of scientific papers, and not merely as homework assignments. This short note intends to provide some advice on how to write mathematical reports, and on scientific writing in general, with a special emphasis on texts concerned with applied or numerical mathematics.

1. BACKGROUND MATERIAL

This note is based on, and to a large degree identical to, a note for the 2017 edition of the class TMA4180, Optimisation I, as well as a note for the course TMA4215, Numerical Mathematics, in 2016 [Kvæ16]. Having been resurrected for the 2023 edition of TMA4180, it is now, in 2024, once again published, though not without its undergoing of some intermediate process of change, with the intent to provide some ideas about scientific writing, particularly in the case of texts in applied or numerical mathematics.

2. ON WRITING

Say something. The most important point when writing a text is to say something. In order to do that, you actually have to *have something to say*. This not only means that you should avoid writing about nothing, which would be hard within the constraints of this project anyway, but also that you should not try to say too many things at once. One of the difficulties you might face during this project is a strict limitation to at most ten pages, while you could easily produce numerical results covering fifteen. It will be much easier to select numerical examples, if you have already decided, what you want to say with them. Also keep in mind that you can include additional numerical examples in the jupyter notebook.¹

Think about your audience. Before you start writing anything, think briefly about the audience you want to reach. It should make a difference, whether you write a text for specialists in numerical mathematics or for first-year students—or somebody who has no mathematical background at all. In particular, this will be important if you have to decide how detailed you have to be, and what you can assume to be “common knowledge.”

For this project, it may help to think that you are writing for fellow students in your study programme who have some background in optimisation (for instance, because they have attended the class in earlier years) but are not familiar with the concrete problem posed in the project. There is thus no need for you to explain what a line search method or a Quasi-Newton method is, or to explain the deeper meaning of KKT conditions or constraint qualifications. However, you need to include an explanation of what the project is about, though this explanation can, and should, be somewhat brief and on point. A general advice, which can be worth

Date: 8th February 2024.

¹Still, I would recommend to be somewhat elective in the jupyter notebook as well and to avoid cluttering it with repetitive examples.

following for all your writings, is to try to formulate everything in such a way that you yourself will still be able to understand it in a few years. The same applies, maybe in even stronger form, to the code you are writing. Try to write it in such a way and *add enough comments* so that you are confident that you will still be able to follow the code if you happen to read or run it some time in the future.

Concerning language. If you are writing the report in English, make sure that your English is actually readable. An analogous statement applies, of course, if you are writing in Norwegian. Spell checkers are valuable tools, and it is a good idea to use one. However, do never rely solely on automated spell checking, as some errors tend to be overlooked by spell checkers, while sometimes new ones are introduced. If you find somebody else to look briefly over your report after you think it is finished, use this opportunity. It is easy to miss errors after one has worked on the same text over a long period of time. This will be even more important when you are writing your master thesis. There it is a good idea to ask a friend or family member well before the deadline if they are willing to glance thoroughly over your thesis.

Rules of writing and good English. There are lots of different guides available on writing, and also on mathematical writing. I can recommend, for instance, the book by Higham [Hig98], or the excellent article by Halmos [Hal70]. Even if you do not find time to read these texts now, try to do so when/before writing your master thesis. However, only ever take all the guides you find as mere guidelines and break any rules outlined there, if you have a good reason for doing so.²

Loops and repetitions. Writing is an iterative process. It might easily happen that you have to rewrite a paragraph or even a section, because it does not fit any more into the rest of the paper. Or, that it turns out that your paper exceeds the strict page limit you have for this project. Do not be afraid of this, as it is part of the writing process. Halmos [Hal70] specifically recommends to “write on the spiral plan,” where one writes and rewrites the sections or chapters in the order 1, 2, 1, 2, 3, 1, 2, 3, 4, . . . : After you have written the first draft of Section 2, you will get new ideas for Section 1 and will realise that some parts need to be formulated or introduced differently, which in turn requires some reformulations in Section 2, and so on. I am not sure that this will be the best approach to this project report, but this plan is surely something to keep in mind for larger texts like your master thesis.

If (or rather: “Since”) you are planning on rewriting many parts of your paper anyway, it does not make much sense to spend a lot time on precise grammar and fancy wordings during the first iterations. Your errors will be rewritten anyway. Only later, when your text is about to converge to a stable solution, should you think about including elaborate and witty, or more commonly less so, formulations. There is a huge *caveat* to this advice, though: It only applies in the effective absence of time constraints. If you start the writing process too close to your deadline, it may be better to just write everything as fast as you can as correct as you can. As usual, constrained optimisation turns out to be more complicated than free optimisation.

Generative AI. Personally, I advise against using large language models like ChatGPT for the composition of the project report. The intention behind the project is not solely to provide an opportunity to play with optimisation algorithms in a mostly harmless but still, as I do hope, not completely uninteresting setting, but also to let you develop your writing and presentation skills before you have to

²“I really want to” counts as a good reason, as long as you do not overuse it.

apply them on your own to your master thesis. An overly reliance on the crutch that modern AI presents risks the arrested development of those important skills.

If you decide not to heed this advice and use generative AI for parts of your project report, state this clearly in a separate section, for instance between the conclusion and the references. This includes the usage of generative AI for writing code, for creating figures, or, heaven forbid, for the derivation of mathematical formulas. State precisely which tools you have used and for which purposes, and be always as specific as possible. As with all tools, the employment of AI happens at your own risk, and it is your responsibility to ensure the correctness of everything that the AI has produced for you.

Collaborative writing. Do write the report in \LaTeX ; this is the state-of-the-art in mathematics, and for good reason. For organising the collaboration on your project report, there are different options:

Since \LaTeX -code is code,³ it works perfectly with version control systems like Git, which you should anyway use for your python code. While I personally prefer this option for my collaborations, it can require some additional organisational overhead, or alternatively an efficient handling of merge conflicts, if different persons need to change the same parts of the text simultaneously.

An often used alternative is *Overleaf*, which allows for real-time editing of \LaTeX -code simultaneously by different people. It also includes some limited version control functionalities that allow you to restore previous versions of your text. As a student at NTNU, you can use it for free after registering with your NTNU e-mail address here: <https://www.overleaf.com/edu/ntnu>.

3. ON WRITING MATHEMATICS

Mathematical writing has its peculiarities, which make mathematical texts different from scientific texts in other disciplines.

Mathematical expressions. One of the most obvious differences between mathematical and non-mathematical texts is the frequent occurrence of mathematical phrases in the former, either within the running text like in the equation $x^2 + bx + c = 0$, or as a separate line like

$$(1) \quad x = -\frac{b}{2} \pm \sqrt{\frac{b^2}{4} - c}.$$

Here are just a few remarks and guidelines concerning their usage in mathematical texts:

- Mathematical symbols should always be used for a reason, usually either in order to make a sentence simpler to read—just try to imagine formulating the equation (1) in plain English—or because it would be difficult or impossible to obtain the same precision without them. However, if a statement can be easily expressed without symbols, there is no reason to introduce them by force. It is, for instance, perfectly fine to say “The square of every non-zero real number is positive.” instead of “If $x \in \mathbb{R}$ and $x \neq 0$, then $x^2 > 0$.”
- Avoid introducing notation if it is not necessary. For instance, in the sentence “Every complex matrix A has an eigenvalue”, naming the matrix A serves no purpose whatsoever. It would be easier and shorter to simply write “Every complex matrix has an eigenvalue” instead.

³ \TeX , on which \LaTeX is based, is actually Turing-complete. Because of severe memory limitations and awful runtimes, it is rarely used for larger computations, though.

- It is good practice, not to begin sentences with mathematical symbols, because they may make the text slightly more difficult to read. Similarly, always try to keep a minimal safety distance of at least one English word between separate mathematical expressions. For instance, do avoid a sentence like “If $x > 0$, $x^2 > 0$,” but rather write “If $x > 0$, then $x^2 > 0$.”
- If a mathematical derivation takes several steps, do not formulate it as a mere list of mathematical expressions, as you would do in a homework assignment or an exam.⁴ Rather, comment briefly on what you are doing in between the formulas. This makes it much easier to follow the argumentation and understand what is happening.
- Mathematical expressions are abbreviations of English (or whatever language you write in) sentences and they should be treated as such. In theory, it should always be possible, if cumbersome, to translate a mathematical expression symbol by symbol into English and obtain a grammatically correct sentence. As a corollary of this, we obtain that mathematical expressions are part of the sentence structure, even if they are written in separate lines. This means in particular that punctuation marks like in (1) are necessary.
- Quantifiers like \forall or \exists and logical operators like \Rightarrow or \Leftrightarrow should be used sparingly. Most of the time, it is better to replace them by the corresponding words, as words tend to be easier to read than symbols.
- Be very careful with footnotes, as they can easily interfere with computations and for instance make equations like $a^2 + b^2 = c^{25}$ almost impossible to read correctly.

Obviously, together with its distant cousins *clearly*, *trivially*, and *it is easy to see*, is one of the most overused and misused words in mathematics. Often it is simply employed as an abbreviation for “I do believe that what I write is true. However, I am too lazy to explain why I think so, and I absolutely do not want to formulate an actual proof.” Do not use the word in this meaning! Also, if you happen to write in your report that something is obvious, make absolutely sure that said something *is actually true*.

We is not that bad. In mathematical writing, it is perfectly fine to use the word “we” both for referring to yourselves and to yourself or yourselves together with the reader, especially if other formulations using, for instance, the passive voice turn out to be much more cumbersome. Other fields may have different conventions, but the mathematical one is, of course, correct.⁶

4. STRUCTURE OF NUMERICAL TEXTS

A typical paper in numerical or applied mathematics has the following general structure:

- Abstract,
- Introduction,
- Main part,
- Numerical experiments,
- Conclusion,
- Bibliography.

⁴In fact, I would, out of pure selfishness, extend the same advice to homework assignments and exams, particularly in subjects like “Optimisation.”

⁵This is not a mistake, but rather a very annoyingly placed reference to a footnote.

⁶Even if you are the sole author of a mathematical text, it is fine to use “we” as a pronoun. There is no need to convince F.D.C. Willard to coauthor the text with you.

I advise to follow this tried and tested structure also in your project report. This does not mean, though, that you should use precisely five sections (+ the abstract) with precisely these headings. The main part and the numerical experiments may be subdivided into several sections, if it makes sense to do so.

Abstract. The abstract should give a very short summary of the main content and the main results of the paper. It should contain enough information to make a potential reader interested in the paper, and should also be seen as a short “advertisement” of the paper. Try to avoid mathematical expressions and also citations in the abstract, unless they are absolutely necessary. An awful and an excellent example of an abstract can be found in [Lan66]. Though not all of the arguments presented there may be applicable to modern times, the general ideas still hold true. I strongly recommend to read thoroughly through that whole paper.

Introduction. The introduction should describe the general setting of the paper and provide an overview of its main results. It is also very common to discuss the general structure of the paper. In scientific papers, and also in your master thesis, the introduction is also the place where you should compare your own results with related work in the same field; in the context of this project, however, this is not required.

In general, it is a good idea to write the introduction only after the rest of the paper is finished⁷ and keep the introduction essentially independent. Also, avoid filling the introduction with technical definitions, but rather postpone as much mathematics as possible to the main part.

Main part. This is the main part of the paper, where “all the interesting mathematics happen.” For this project, this will be the place where you define all the functions and the optimisation problem, discuss its properties, and also describe the numerical method used for its solution. This is *not* the place, where you would usually perform numerical experiments, though. Often, the main part will not just consist of a single section, but rather be split into several sections, each devoted to a particular sub-problem. Use your common sense to find a reasonable structure.

Try to write short, but ensure that everything you say is correct. You will probably have to introduce some mathematical notation, but do so sparingly and try to keep your notation consistent (use similar symbols for similar objects).

Cite all your sources. Feel free to use all available resources, but never copy anything word by word and always include citations in your paper.⁸

Numerical experiments. This is the section, where you describe the numerical results you obtain by applying the methods described in the main part. Make sure that it is, in principle, possible to reproduce your experiments just from your descriptions in the paper. This means that you should provide all the details of the algorithms you have used, including all parameters, and also the settings in the specific experiments (e.g. configuration of the tensegrity structure, parameters of

⁷Or rather: after the first version of the rest of the paper is finished. You might need to revise the other sections after finishing the introduction.

⁸Similar to other fields, mathematics uses the convention that “common knowledge” need not be cited. There is, however, no general agreement as to what constitutes common knowledge. Pythagoras’ theorem certainly is; within the context of this project, KKT conditions are, as are Quasi-Newton or non-linear CG methods; an inverse function theorem for weakly differentiable maps with respect to the analytic topology of locally Euclidean metrisations of infinitely differentiable Riemannian manifolds most probably are not and require a citation in order to avoid any accusations of plagiarism.

For this project, you can assume that everything we have discussed in class is common knowledge and warrants no citation, and the same holds for the content of your basic mathematics and numerics classes. Also, you don’t have to cite the project description in your report.

the structure and also the numerical algorithm, starting values of the iteration, . . .). Be careful that your numerical results do not contradict the results one obtains with your attached code. Be particularly careful with last-minute revisions of either your document or your code.

Feel free to use figures for the presentation of the results; in particular convergence rates often look far more convincing if presented with figures than with tables. If you do so, make sure that your figures are readable (both the plots themselves and the annotation). Also, add captions to the figures that explain their content in a concise manner.

Conclusion or summary. In this part, you should review once again the results obtained in the paper, and for instance discuss whether the numerical results were consistent with your theoretical predictions or not, and whether your method showed some “interesting” behaviour (e.g. failure in certain cases or, conversely, situations where it worked better than expected). In your master thesis, this would also be the place where you can discuss possible open (or new) questions and propose new directions of research.

Bibliography. Make sure that you cite all sources you have used during your project (don’t cite the project description). As discussed above, it is not necessary to provide references for “common knowledge”, that is, basic mathematical results that are well known to you and also to your intended audience.

I advice to use BibTeX for setting up the bibliography, as it works very well together with L^AT_EX and makes it easy to ensure a uniform style for the references. BibTeX entries for almost all mathematical articles and books published in the last hundred years can be found at <https://mathscinet.ams.org/mathscinet/>. You will need an NTNU IP address in order to access this database, though.

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