



Contact during exam:

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EXAM IN SIF5032 CODING THEORY

English

Saturday 25. May 2002

Time: 0900-1400

Permitted aids:

approved calculator

all printed or written aids

The grades are posted in week 24.

Problem 1

Let C be the binary code given by the generatormatrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

- Find the minimum distance of the code?
- A received vector $\underline{v} = (1111100x)$ has at most one error and one erasure.
Find the codeword that was sent.
- Let \tilde{C} be a code where the elements are 3×8 -matrices with rows in C , and such that row 3 is the (binary-)sum of row 1 and row 2.
Then \tilde{C} can correct ≤ 3 errors. Explain why and how?

Problem 2

Let α be a primitive element in $GF(32)^*$, with $\alpha^5 + \alpha^2 + 1 = 0$.

- a) Show that $m(x) = x^5 + x^4 + x^3 + x^2 + 1$ is the minimal polynomial in $GF(2)[x]$ of α^3 .

A BCH-code C is defined by the conditions

$$f_{\underline{c}}(\alpha) = f_{\underline{c}}(\alpha^3) = 0, \quad \deg f_{\underline{c}} < 32.$$

- b) Find a generator polynomial for C .
Show that the minimum distance of C is 5.

- c) Assume a vector \underline{r} is received, with

$$f_{\underline{r}}(\alpha) = 1, \quad f_{\underline{r}}(\alpha^3) = \alpha^7 + 1.$$

If at most two errors have occurred, where in \underline{r} are the errors?

Problem 3

Let $g(x) = 1 + x^2 + x^4 + x^5$ in $GF(2)[x]$, and let $C = \langle g(x) \rangle$ be a binary cyclic code of length 15.

- a) Compute $(x^{15} + 1) : g(x)$. Find a parity-check matrix for C .
- b) Find the minimum distance of C ?
- c) Assume you have received a vector where either one digit is wrong or two neighbouring digits have changed place. How can you find the codeword that was sent?

Problem 4

- a) For which values of k is there a binary $[13, k, 3]$ -code?
- b) For which values of k is there a binary *cyclic* $[13, k, 3]$ -code?
- c) For which positive values of r is there a binary $[13 + r, 9 + r, 3]$ -code? Find (the generatormatrix of) a code with these parameters and biggest possible value of r .