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Contact during exam:
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EXAM IN TMA 4185 Coding theory

English
Tuesday May 19, 2009
Time: 0900-1300

Permitted aids: All printed and written aids. Approved calculator.

Problem 1 Let C be the binary code generated by:
$\{11101000, 10101011, 01101110, 10011000, 10110101\}$.

a) Find a parity check matrix for C. What is the length, dimension and distance of C?

b) Is C cyclic?

Problem 2 We look at binary convolutional codes.

a) Let C be a convolutional code with generator $g(x) = 1 + x + x^3$. Draw the diagram for the shift register corresponding to C. Assume all registers are initially zero. Compute the output for the input sequence: 01110010000....

b) Let C' be the convolutional code with generators $g'_1(x) = 1 + x^2$ and $g'_2(x) = x$. Draw the state machine for C'. Is 001001001111100000... a codeword?

Problem 3 Does there exist a linear or non-linear code of length $n = 10$ and minimum distance $d = 7$ over $\mathbb{F}_3$ with 81 codewords?

Problem 4 Let C be the RS code over $\mathbb{F}_{16}$, constructed using the polynomial $1 + x + x^4$, with length 15, designed distance $\delta = 6$ and defining set $T = \{1, 2, 3, 4, 5\}$. Then $\alpha = \alpha^4 + 1$ is a primitive 15th root of unity. The construction of the field is given below.

a) Find a generator polynomial for C. What is the distance and dimension of C?

b) Suppose the word $y(x) = x^7 + \alpha^8 x^5 + \alpha^{13} x^4 + \alpha^3 x^3 + \alpha^{10} x^2 + \alpha x + 1$ is received. Decode $y(x)$.

Problem 5 Let C be a binary linear code. Show that all or exactly half of the codewords have even weight.

Problem 6 Show that $\mathrm{A}_2(8, 5) = 4$. (Hint: Try to construct such a code.)

Construction of $\mathbb{F}_{16}$ with the polynomail $x^4 + x + 1$

Polynomer	α^i
0	
1	α^0
α	α^1
α^2	α^2
α^3	α^3
$\alpha + 1$	α^4
$\alpha^2 + \alpha$	α^5
$\alpha^3 + \alpha^2$	α^6
$\alpha^3 + \alpha + 1$	α^7
$\alpha^2 + 1$	α^8
$\alpha^3 + \alpha$	α^9
$\alpha^2 + \alpha + 1$	α^{10}
$\alpha^3 + \alpha^2 + \alpha$	α^{11}
$\alpha^3 + \alpha^2 + \alpha + 1$	α^{12}
$\alpha^3 + \alpha^2 + 1$	α^{13}
$\alpha^3 + 1$	α^{14}