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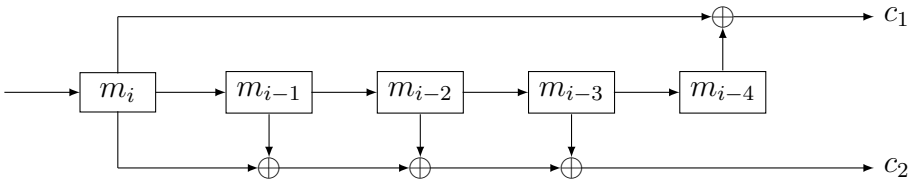
EXAM IN TMA4185 CODING THEORY

English
Thursday, May 31, 2012
Time: 15.00–19.00
Any printed or hand-written material and an approved simple calculator are allowed.
Solve all four (4) problems. Show all your work and justify all answers.

Problem 1 A binary convolutional code $\mathcal{C}_1$ is implemented using the shift register below.



Find a generator matrix for $\mathcal{C}_1$ and the interleaved encoding of the message $m = 0101010111$.

Problem 2 This problem is about ternary cyclic codes of length 8.

a) How many such codes exist?

b) Give the defining set of such a code with 243 codewords.

c) Give the generating polynomial of such a code which is BCH with designed distance 5. [Hint: $x^2 + x + 2$ is irreducible over $\mathbb{F}_3$.]

Problem 3 The matrix $G_2 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{bmatrix}$ generates a ternary linear code $\mathcal{C}_2$.

a) Show that the code is self–dual and find its minimum distance $d(\mathcal{C}_2)$.

b) A message was first encoded using G_2 and then transmitted. If the received vector had syndrome $[1\ 2]^T$, what was the error vector, if any? Was it unique?

c) Determine $A_3\,(4, d(\mathcal{C}_2))$.

d) Modify $\mathcal{C}_2$ to obtain a different code of the same length and minimum distance at least $d(\mathcal{C}_2) - 1$.

Problem 4 The ISBN is a length 10 error–detecting code whose every codeword $(c_1 c_2 \ldots c_{10})$ satisfies

$$\sum_{i=1}^{10} i c_i \equiv 0 \mod 11 \ ,$$

where the first nine digits $c_1, \ldots, c_9 \in \mathbb{Z}_{10}$ and $c_{10} \in \mathbb{Z}_{11}$ (with 10 represented as X).

Is it linear? Show that it can detect the most frequent errors people make when typing, namely single errors and adjacent digit transpositions (i.e. the swapping of digits c_i and c_{i+1}).

[End of paper]