



- 1 A physical quantity F is dependent on the four (non-zero) physical quantities a, b, c, d . For two physical dimensions x and y , the dimensions of the physical quantities are:

$$[F] = x^2/y, \quad [a] = y, \quad [b] = 1/(xy), \quad [c] = y/x^2, \quad [d] = x/y,$$

- a) Two valid conclusion of the Buckingham Pi Theorem applied to the above case, are given by

$$F = \frac{1}{c} \varphi(a^2bd, acd^2), \quad F = \frac{d}{ab} \phi(c^2d^3/b, ad\sqrt[3]{bc}),$$

for functions φ and ϕ . Although seemingly different, prove that these conclusions are the same; that is, show both that for every ϕ , there is a φ , so the two expressions for F coincides, and vice versa.

Hint: For notational convenience, substitute $\pi_1 := a^2bd$, $\pi_2 := acd^2$, $\tilde{\pi}_1 := c^2d^3/b$ and $\tilde{\pi}_2 := ad\sqrt[3]{bc}$. To show the first part (for every ϕ , there is a φ) it suffices to show that the RHS of the equation $\varphi(\pi_1, \pi_2) = \frac{cd}{ab} \phi(\tilde{\pi}_1, \tilde{\pi}_2)$ can be written solely in terms of π_1 and π_2 .

- b) Suppose we learn that F also is dependent on the physical quantity e , with dimensions $[e] = x$. Consider the following expression

$$F = \frac{e^2}{a} \psi\left(abe, \frac{ce^3}{a^2d}, \frac{a^5b^3d}{c}\right),$$

for a function ψ . Why is this expression *not* a valid conclusion of the Buckingham Pi Theorem reapplied to cover the new information?

- 2 (Problem 4.1.3 p. 51 in Krogstad)
What are the possible choices of core variables (dimensional independent variables) from the set $\{R_1, \dots, R_6\}$ if the dimension matrix looks like:

	R_1	R_2	R_3	R_4	R_5	R_6
F_1	1	1	-1	0	2	2
F_2	-2	-1	1	0	-3	-2
F_3	0	1	0	1	0	2

3 (Problem 4.1.11 p. 53 in Krogstad)

The necessary force (F) to keep a ship at a constant speed (U) depends on its shape, primarily the length (L), width (W), and its depth into the water (D). In addition, the water density, ρ , the water viscosity, ν , and the acceleration of gravity, g , play a part. Use dimensional analysis to find an expression for the force which includes the two most famous dimensionless numbers in ship design:

$$\text{Froude number: } Fr = U/\sqrt{Lg},$$
$$\text{Reynolds number: } Re = LU/\nu.$$

Ideally, a scale model¹ of the ship may be tested experimentally in water by keeping the dimensionless numbers for the model equal to those of the original ship. Is this really possible? Hints: $[F] = \text{kgm/s}^2$, $[\rho] = \text{kg/m}^3$, $[\nu] = \text{m}^2/\text{s}$, $[g] = \text{m/s}^2$.

4 We consider a rubber band which may be stretched to many times its original length l_0 . The rubber band has a "density" ρ which we measure in mass per unit length, that is, kg/m. How does the density ρ vary when we stretch the band to a length l from its original length l_0 and density ρ_0 ?

After stretching the band more than twice its original length, we pluck the band like a guitar string. This experiment shows, somewhat unexpectedly, that the pitch (=frequency ω) remains almost constant when we vary the length (*try it!*). However, when stretching the band up towards its breaking limit, the frequency increases somewhat. The force F required for stretching the band to length l is proportional to $l - l_0$ over most of the range, that is, $F = F_0 \frac{l-l_0}{l_0}$, where F_0 is a constant.

Use dimensional analysis to explain the behaviour of the frequency.

Hint: Assume first that $\omega = f(l, \rho, F)$, apply dimensional analysis, and then introduce the expression of the density as a function of l, l_0 and ρ_0 .

¹A *scale model* is a model of the ship with the same geometric shape, but with a smaller size (say, $L = 1\text{m}$ for the model, compared to 100m for the original ship).