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TMA4195
Mathematical Modelling
Autumn 2018

Exercise set 4

1 (Exercise 2a p. 298 in Lin & Segel)

Find leading order outer, inner and uniform solutions to the following problem.
(Assume that ε is small and positive and that the boundary layer is at $x = 0$.)

$$\begin{aligned}\varepsilon y'' + (1+x)y' + y &= 0, \\ y(0) &= 0, \\ y(1) &= 1.\end{aligned}$$

2 (Exercise 2b p. 298 in Lin & Segel)

Find leading order outer, inner and uniform solutions to the following problem.
(Assume that ε is small and positive and that the boundary layer is at $x = 0$.)

$$\begin{aligned}\varepsilon y'' + y' + y^2 &= 0, \\ y(0) &= \frac{1}{4}, \\ y(1) &= \frac{1}{2}.\end{aligned}$$

3 (Exercise 5 p. 299 in Lin & Segel)

Use singular perturbation theory to obtain leading order outer, inner, and uniform solutions of the problem

$$\varepsilon u'' - (2 - x^2)u = -1, \quad u(-1) = u(1) = 0.$$

REMARK. It is sufficient to solve the differential equation on $(0, 1)$ subject to the boundary conditions $u'(0) = 0$ and $u(1) = 0$. Why?