Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4195 Mathematical Modelling Autumn 2019

Exercise set 4

$\boxed{1}$ (Exercise 2a p. 298 in Lin & Segel)
Find leading order outer, inner and uniform solutions to the following problem. (Assume that ε is small and positive and that the boundary layer is at $x = 0$.)

$$\begin{aligned} \varepsilon y'' + (1+x)\,y' + y &= 0, \\ y(0) &= 0, \\ y(1) &= 1. \end{aligned}$$

$\boxed{2}$ (Exercise 2b p. 298 in Lin & Segel)
Find leading order outer, inner and uniform solutions to the following problem. (Assume that ϵ is small and positive and that the boundary layer is at $x = 0$.)

$$\begin{aligned} \epsilon y'' + y' + y^2 &= 0, \\ y(0) &= \frac{1}{4}, \\ y(1) &= \frac{1}{2}. \end{aligned}$$

$\boxed{3}$ (Exercise 5 p. 299 in Lin & Segel)
Use singular perturbation theory to obtain leading order outer, inner, and uniform solutions of the problem

$$\epsilon u'' - (2 - x^2)u = -1, \qquad u(-1) = u(1) = 0.$$

REMARK. It is sufficient to solve the differential equation on $(0, 1)$ subject to the boundary conditions $u'(0) = 0$ and $u(1) = 0$. Why?