

## Key Concepts

- Background
  - Eigenvalues and eigenvectors
  - Jordan canonical form
  - Normal matrices
  - Hermitian matrices
  - Orthogonal matrices
  - Unitary matrices
  - Skew-symmetric matrices
  - Positive definite and diagonal dominant matrices
  - Norms of vectors, matrix-norms, operator norms
  - Inner products, inner product norms
  - Rank and nullity, kernel and range of a matrix
  - Existence and uniqueness of solutions of linear systems of algebraic equations
  - Similarity transformations
  - Convergence of fixed point iteration methods for linear systems of equations
- Spectral radius, Gershgorin theorem
- Perturbation analysis for linear systems and eigenvalues. Condition numbers
- Projection methods: error projection and residual projection. The optimality results
- Steepest Descent, Minimal Residual. Krylov space methods: FOM, GMRES, Lanczos, Conjugate Gradient
- Convergence of GMRES and CG
- Implementation of Least square problem for GMRES
- Preconditioning
- Multigrid. Smoothers, prolongation and restriction operators.
- Householder and Givens transformations
- QR factorization
- Schur Factorization (including real Schur factorization)
- Unsymmetric eigenvalue problem: power method, inverse iteration, shifted inverse iteration, orthogonal iteration, QR-iteration, single shift QR
- Implicit Q-Theorem
- SVD, reduced SVD, best approximation with truncated SVD expansion
- Practical implementation of the SVD, the Golub and Kahan method.