Course content

In this course, we will develop and analyse schemes for solving PDEs. We will (more or less) follow the following procedure:

1. Given a problem (PDE + Ω + IC/BC)
2. Construct a scheme
3. Implement and test the scheme
4. Do an error analysis
5. Verify theoretical results numerically
6. Explain and if possible circumvent unexpected behaviour

Numerical schemes:

- Finite difference methods (FDM)
- Finite element methods (FEM)

Linear PDEs in 2D

The main focus is on linear PDEs in 2D,

$$au_{xx} + 2bu_{xy} + cu_{yy} + du_x + eu_y + gu = f \qquad \text{in } \Omega,$$

where $\Omega \in \mathbb{R}^2$ is the domain, $u : \Omega \to \mathbb{R}$ is the solution and $a, b, \ldots, f : \Omega \to \mathbb{R}$ are coefficients (also called data).
These are classified by:

Classification		Model problems	
$b^2 - ac < 0$,	elliptic	$u_{xx} + u_{yy} = f$,	Poisson equation
$b^2 - ac = 0$,	parabolic	$u_t = u_{xx}$,	Heat equation
$b^2 - ac > 0$,	hyperbolic	$u_{tt} = u_{xx}$,	Wave equation

Boundary/initial conditions are needed for unique solutions:

$$\begin{aligned} \text{Initial conditions (IC):} &\quad u \text{ given at } t = 0, \\ \text{Boundary conditions (BC):} &\quad u \text{ or } \partial u_n \text{ given at } \partial\Omega. \end{aligned}$$