NTNU

TMA4215 Numerical Mathematics — Fall 2016

Problem Set 10

1 Determine the values of (a, b, c) such that

$$S(x) = \begin{cases} S_0(x) = x^3 & x \in [0,1) \\ S_1(x) = \frac{1}{2}(x-1)^3 + a(x-1)^2 + b(x-1) + c & x \in [1,3] \end{cases}$$

is a cubic spline. Is it a natural cubic spline?

2 Given the data set:

x	1.2	1.5	1.6	2.0	2.2
$f(x)$	0.4275	1.139	0.8736	−0.9751	−0.1536

Find the linear spline $L(x)$ interpolating the data set, and give the value for $L(1.8)$.

3

a) Write down the 4 cubic Bernstein polynomials, $b_{3,i}(t)$, $i = 0, 1, 2, 3$.

b) Given the control points

$$\mathbf{P}_0 = (0,0), \qquad \mathbf{P}_1 = (1,2), \qquad \mathbf{P}_2 = (2,-1), \qquad \mathbf{P}_3 = (1,0).$$

Write up and plot the corresponding Bezier curve,

$$\mathbf{B}(t) = \sum_{i=0}^{3} \mathbf{P}_i b_{3,i}(t).$$

Prove that the straight line between the first two control points is tangential to the curve in $\mathbf{P}_0$, similar is the straight line between the last two point tangential to the curve in $\mathbf{P}_3$.

c) Assume that you know the value of a function $f(x)$ and its derivative $f'(x)$ for two values of x, say x_0 and x_1, with $h = x_1 - x_0 > 0$. Let $x = x_0 + th$ and let

$$B(x) = B(x_0 + th) = \sum_{i=0}^{3} a_i b_{3,i}(t).$$

Find a_i, $i = 0, 1, 2, 3$ such that

$$B(x_i) = f(x_i), \qquad B'(x_i) = f'(x_i), \qquad i = 1, 2$$

where $B' = dB/dx$.