Norwegian University of Science and Technology
Institute of mathematics

TMA4220 Finite Element Method Fall 2016

Exercise set 3

1 If you are not familiar with the Lebesgue spaces $L^p(\Omega)$ and the Sobolev spaces $H^p(\Omega)$, you should read section 2.3.1 and 2.4.0-2.4.2

Are the following statements *true or false*?

a) The set $S = \{v \in C^0(0,1) : v(\frac{1}{2}) = 1\}$ is a linear (vector) space

b) For $V = H_0^1(0,1), F(v) = \int_0^1 xv \, \mathrm{d}x$ is a linear functional.

c) The only v in $H^1(\Omega)$ for which $|w|_{H^1(\Omega)}$ (the H^1 semi-norm) is zero is $v = 0$.

d) The function $v = x^{3/4}$ is in $L^2(0,1)$; in $H^1(0,1)$; in $H^2(0,1)$.

e) For $v = e^{-10x}$, $|v|_{H^2(0,1)} = |v|_{H^1(0,1)}$.

2 For each of the following problem, *(i)* find a weak formulation of the PDE and *(ii)* choose an appropriate test/trial space V where u and v live. Show that the conditions of the Lax-Milgram theorem are satisfied, thus proving that there exist a unique (weak) soltuion of the PDE, by proving $a(\cdot,\cdot)$ is *(iii)* continuous and *(iv)* coercive. Assume $\Omega \subset \mathbb{R}^2$ is an open, bounded, connected domain and $f \in L^2(\Omega)$.

a) The biharmonic equation:

$$\begin{aligned} \nabla^4 u &= f \quad \text{in } \Omega \\ u &= 0 \quad \text{on } \partial\Omega \\ \frac{\partial u}{\partial n} &= 0 \quad \text{on } \partial\Omega \end{aligned}$$

b) The Convection-Diffusion equation

$$\begin{aligned} -\nabla^2 u + a \cdot \nabla u &= f \quad \text{in } \Omega \\ u &= 0 \quad \text{on } \partial\Omega \end{aligned}$$

where $a = a(\mathbf{x} : \Omega \to \mathbb{R}^2$ is a given differentiable function, the *velcotiy field*, satisfying $\nabla \cdot a(\mathbf{x}) = 0$ for all $\mathbf{x}$.