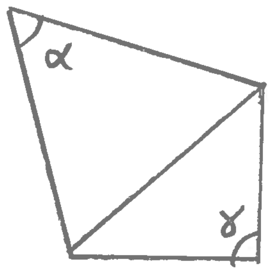
Solution Set V

#1

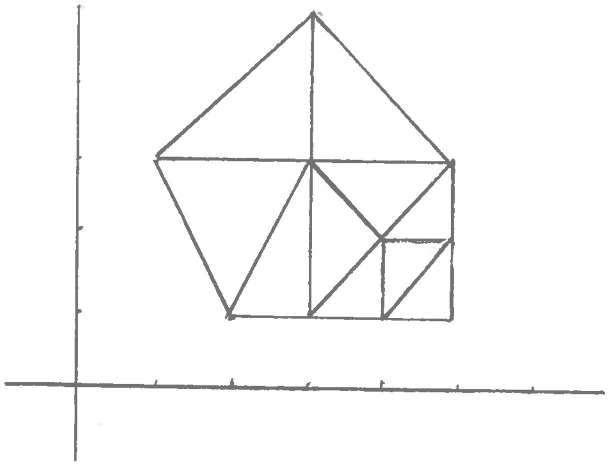
a) <u>This is Delaunay</u> since for every pair of neigbouring triangles



$\alpha + \gamma < 180$

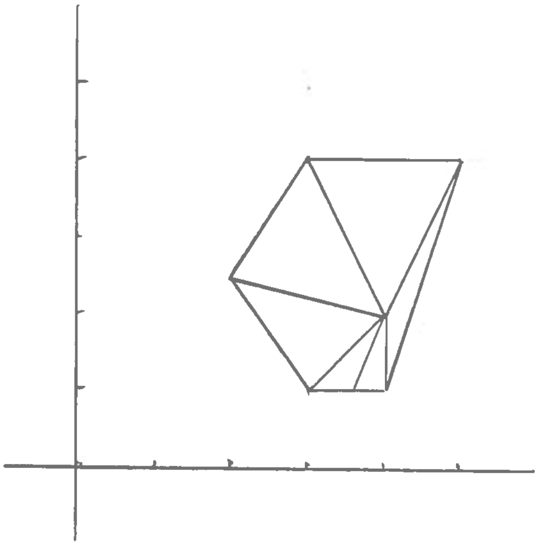
No diagonal flips are possible.

b) <u>Not Delaunay</u> due to the hanging node at (4, 2)



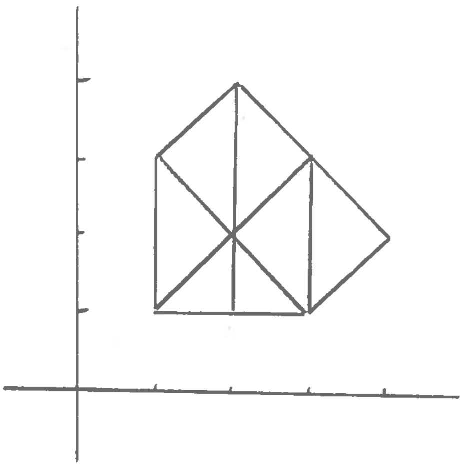
Fixed triangulation

c) **Not Delaunay** since the line (3.5, 1) - (3, 4) can be flipped



Fixed triangulation

d) **Not Delaunay** since the domain is not the convex hull



Fixed Triangulation

e) **This is Delaunay**

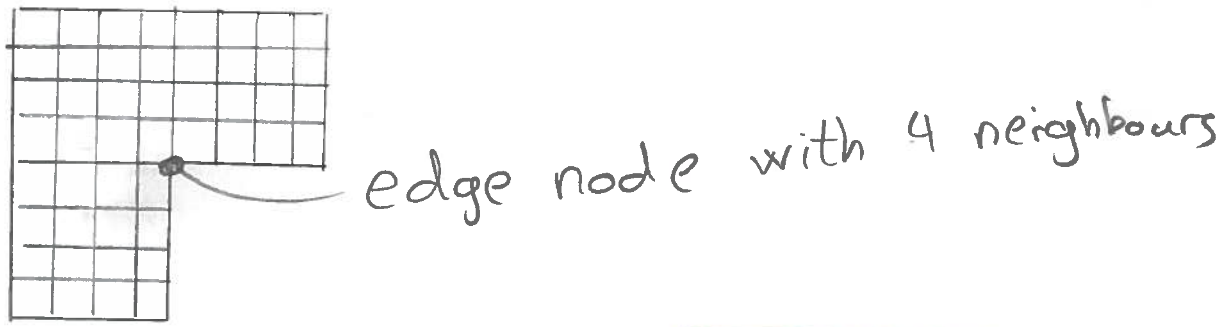
f) **This is Delaunay**

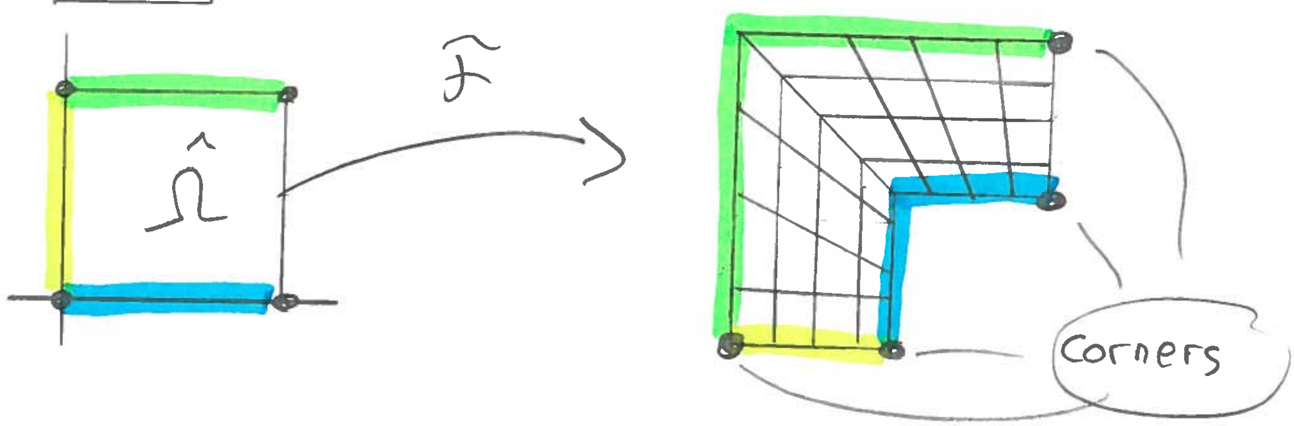
#2

A structured mesh needs 4 corners and 4 sides to map the unit square onto the mesh. A consequence is that

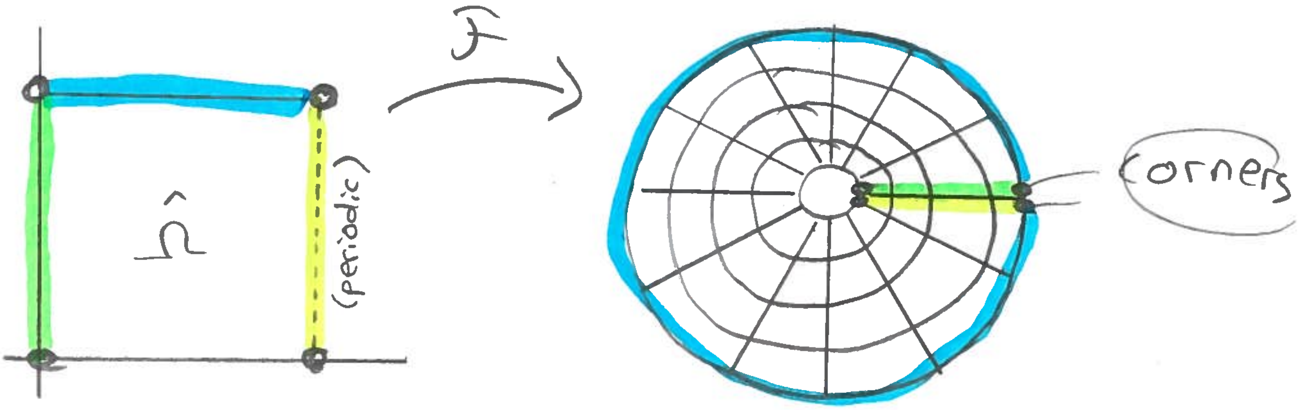
1. all interior nodes have 4 neighbours
2. all edge nodes have 2 or 3 neighbours

a) Not a structured mesh

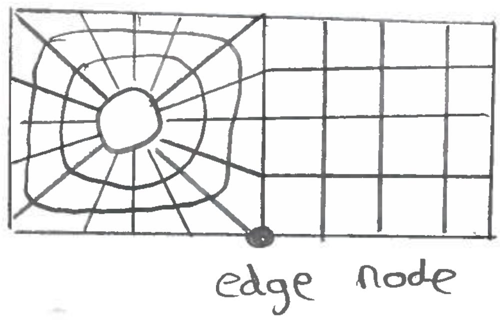


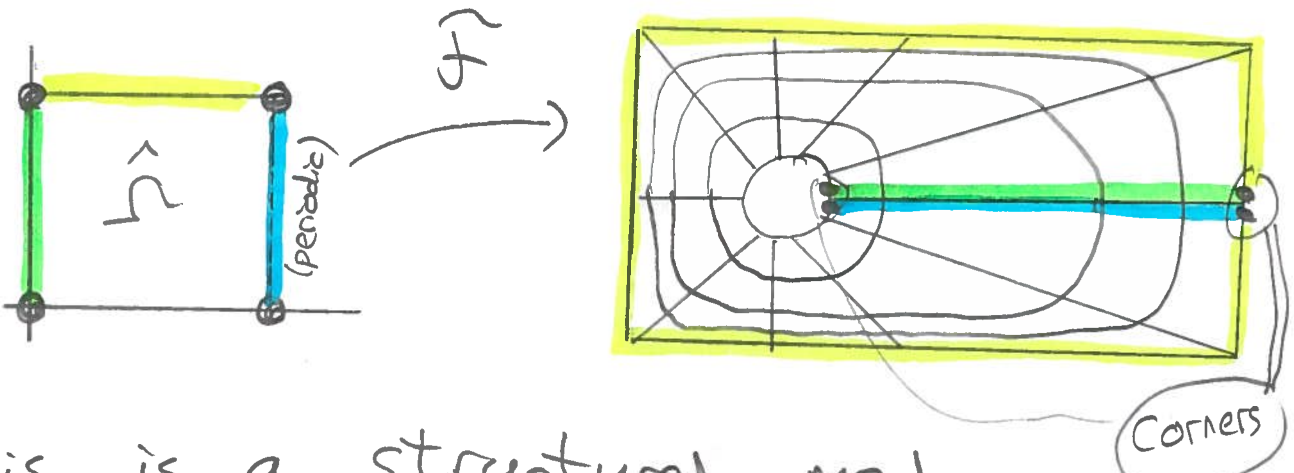


b) This is a structured mesh

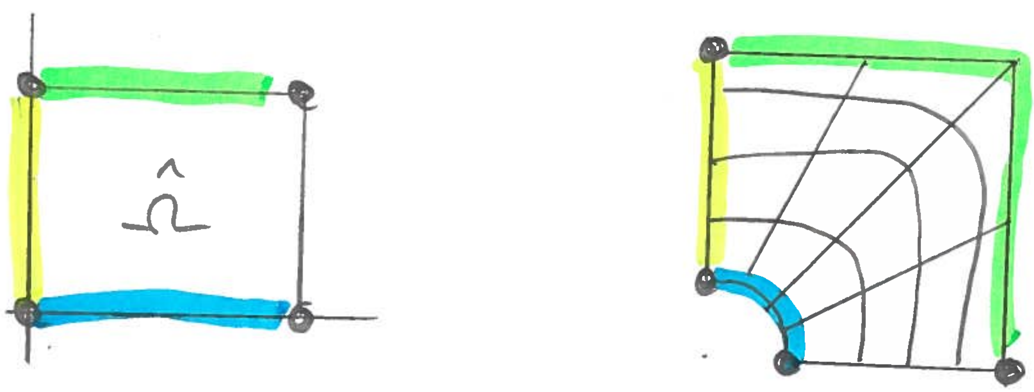


c) Not structured



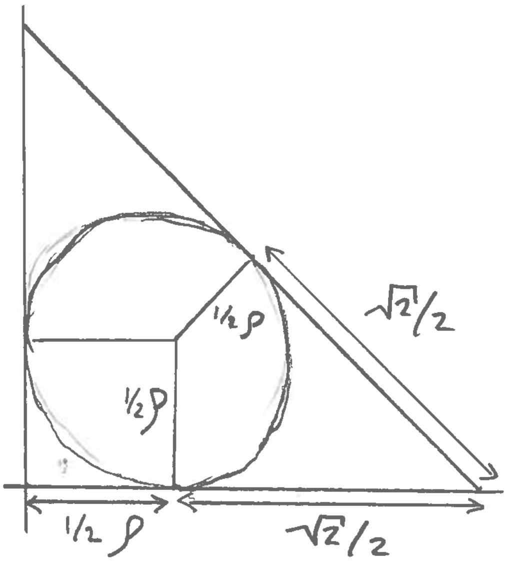


d) This is a structured mesh



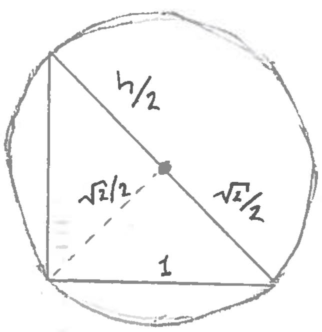
#3

a)



$$1 = \tfrac{1}{2}\rho + \frac{\sqrt{2}}{2}$$

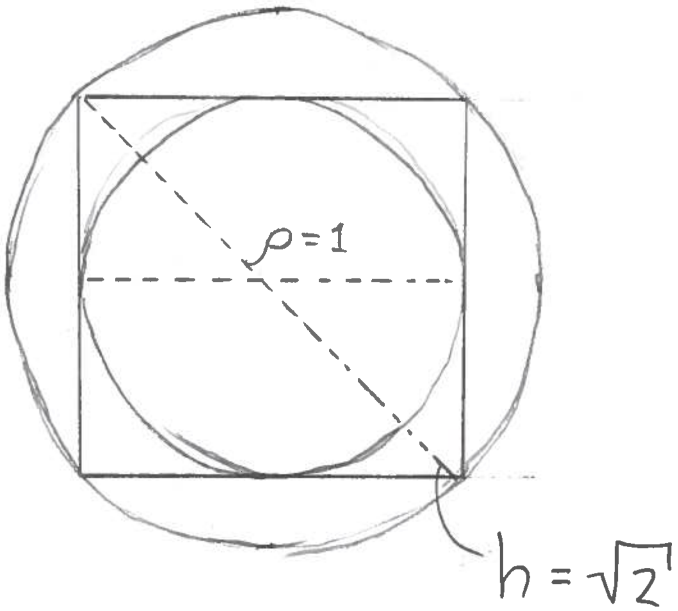
$$\underline{\underline{\rho = 2 - \sqrt{2}}}$$



$$\frac{h}{2} = \frac{\sqrt{2}}{2}$$

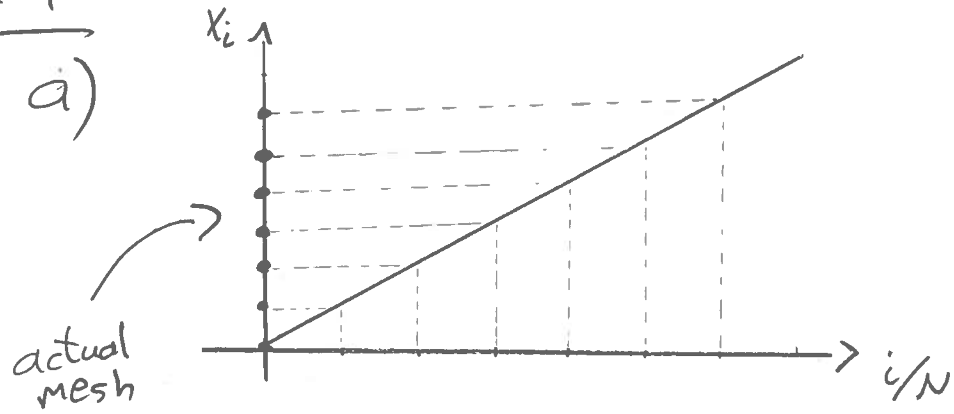
$$\underline{\underline{h = \sqrt{2}}}$$

b)



#4

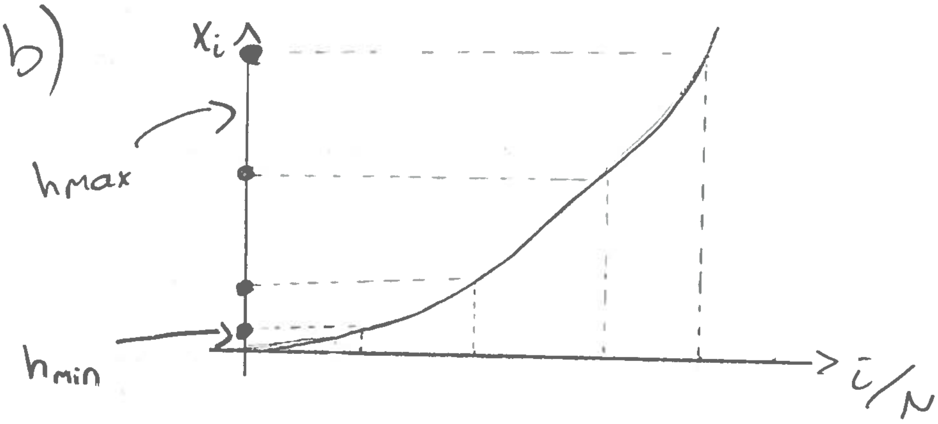
a)



all h equal $\quad \dfrac{\min h}{\max h} = \dfrac{h}{h} = \underline{1 = \tau}$

This is quasi-uniform

b)



$$h_{max} = x_n - x_{n-1} = \frac{0.5n^2 - 0.5(n-1)^2}{n^2}$$

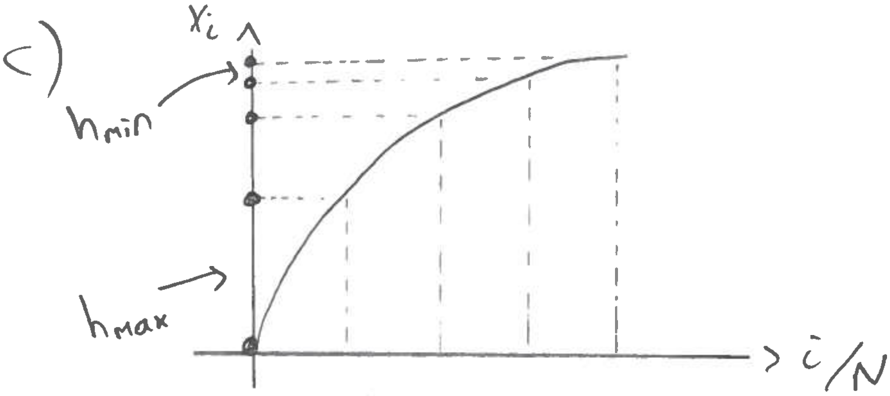
$$h_{min} = x_1 - x_0 = \frac{0.5 - 0}{n^2}$$

$$\lim_{n\to\infty} \frac{h_{min}}{h_{max}} = \lim_{n\to\infty} \frac{\frac{\frac{1}{2}n^2 - \frac{1}{2}(n-1)^2}{\cancel{n^2}}}{\frac{\frac{1}{2}}{\cancel{n^2}}} = n^2 - (n-1)^2$$

$$= 2n - 1 \to \infty$$

Not quasi-uniform

c)



$$h_{max} = x_1 - x_0 = \sin\left(\frac{\pi}{2n}\right)$$

$$h_{min} = x_n - x_{n-1} = 1 - \sin\left(\frac{(n-1)\pi}{2n}\right)$$

$$\lim_{n\to\infty} \frac{h_{min}}{h_{max}} = \lim_{n\to\infty} \frac{1-\sin\left(\frac{(n-1)\pi}{2n}\right)}{\sin\left(\frac{\pi}{2n}\right)}$$

$$\text{l'hôpital} \Rightarrow \lim_{n\to\infty} \frac{-\frac{\pi}{\cancel{2}n^2}\cos\left(\frac{(n-1)\pi}{2n}\right)}{-\frac{\pi}{\cancel{2}n^2}\cos\left(\frac{\pi}{2n}\right)} \longrightarrow 0$$

No $\tau > 0$ exist
not quasi-uniform

d) $h_{max} = \sin\left(\frac{\pi}{4n}\right)$

$h_{min} = 1 - \sin\left(\frac{(n-1)\pi}{4n}\right)$

$$\lim_{n\to\infty} \frac{h_{min}}{h_{max}} = \lim_{n\to\infty} \frac{1-\sin\left(\frac{(n-1)\pi}{4n}\right)}{\sin\left(\frac{\pi}{4n}\right)}$$

$$\text{l'hôpital} \Rightarrow \lim_{n\to\infty} \frac{-\frac{\pi}{4n^2}\cos\frac{(n-1)\pi}{4n}}{-\frac{\pi}{4n^2}\cos\left(\frac{\pi}{4n}\right)} = \frac{\cos\left(\frac{\pi}{4}\right)}{\cos(0)} = \frac{\sqrt{2}}{2} = \tau$$

Quasi-uniform