

1. Prove that a closed subspace of a reflexive space is reflexive. (*Hint:* Let  $X$  be reflexive and  $Y$  a closed subspace. If  $\eta \in Y''$ , consider the functional  $\xi$  on  $X'$  defined by  $\xi(f) = \eta(f|_Y)$ , where  $f|_Y$  is the restriction of  $f$  to  $Y$ . Use the fact that  $X$  is reflexive to represent  $\xi$  by a vector  $x \in X$ . Then use the Hahn-Banach theorem to show that  $x$  must in fact belong to  $Y$ .)