TMA4240 F19
25.10.2016

Intervaller: Rolle til kritisk verdi

$X_1, X_2, \ldots, X_n$ u.i.f $N(\mu, \sigma^2)$

$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$

$(1-\alpha)\cdot 100\%$ KI for μ, når σ ukjent

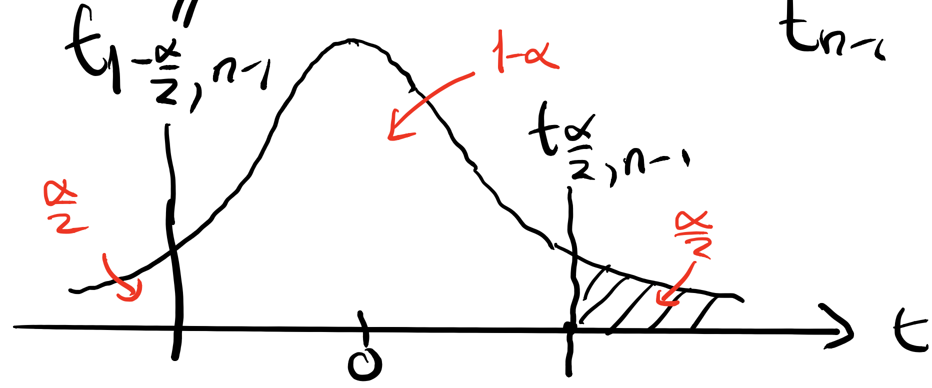
$$\left[\bar{x} \pm t_{\frac{\alpha}{2}, n-1}\frac{s}{\sqrt{n}}\right]$$

$$\boxed{\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)}$$

$$\boxed{\frac{S^2(n-1)}{\sigma^2} = V \sim \chi^2_{n-1}} \rightarrow \frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t_{n-1}$$

$-t_{\frac{\alpha}{2}, n-1}$ (symmetri rundt 0)

$\parallel$



$$P(T_{n-1} > t_{\frac{\alpha}{2}, n-1}) = \frac{\alpha}{2}$$

$$(1-\alpha)\cdot 100\% = 95\% \Rightarrow \alpha = 0.05$$

$$\frac{\alpha}{2} = 0.025$$

$z_{0.025} = 1.96$

$$t_{\frac{\alpha}{2}, 29} = 2.045$$

$$t_{0.025, 29}$$

$$(1-\alpha)\cdot 100\% = 90\% \Rightarrow \alpha = 0.1$$

$$\frac{\alpha}{2} = 0.05$$

$z_{0.05} = 1.645$

$$\left.\begin{array}{l} t_{\frac{\alpha}{2}, 29} \\ t_{0.05, 29} \end{array}\right\} 1.699$$

$$(1-\alpha)\cdot 100\% = 99\% \Rightarrow \alpha = 0.01$$

$$\frac{\alpha}{2} = 0.005$$

$z_{0.005} = 2.576$

$$t_{\frac{\alpha}{2}, 29} \nearrow 2.756$$

$$t_{0.005, 29}$$

NB: $P(\underset{Z}{T} > t_{\text{tall}}) = \text{tall}$ i tabellene

Prediksjonsintervall [9.6] (PI)

$X_1, X_2, \ldots, X_n$ u.i.f $N(\mu, \sigma^2)$ ← σ^2 ukjent

X_0 uavhengig av $X_1, \ldots, X_n$

↑
ny observasjon $\qquad X_0 \sim N(\mu, \sigma^2)$

KI: intervall vi har stor tillit til at sann μ ligger.
↑ 95%

PI: Intervall der vi har stor tillit til at fremtidig observasjon X_0 ligger.

Utlede PI (σ^2 ukjent):

Triks: ser på fordelingen til $(X_0 - \bar{X})$

1) $(X_0 - \bar{X})$ er normalfordelt

2) $E(X_0 - \bar{X}) = E(X_0) - E(\bar{X})$

$$= \mu - \mu = 0$$

3) $\mathrm{Var}(X_0 - \bar{X}) \underset{\text{unabh.}}{=} \mathrm{Var}(X_0) + (-1)^2\,\mathrm{Var}(\bar{X})$

$$= \underbrace{\mathrm{Var}(X_0)}_{\sigma^2} + \underbrace{\mathrm{Var}(\bar{X})}_{\frac{\sigma^2}{n}} = \sigma^2\left(1 + \frac{1}{n}\right)$$

4)

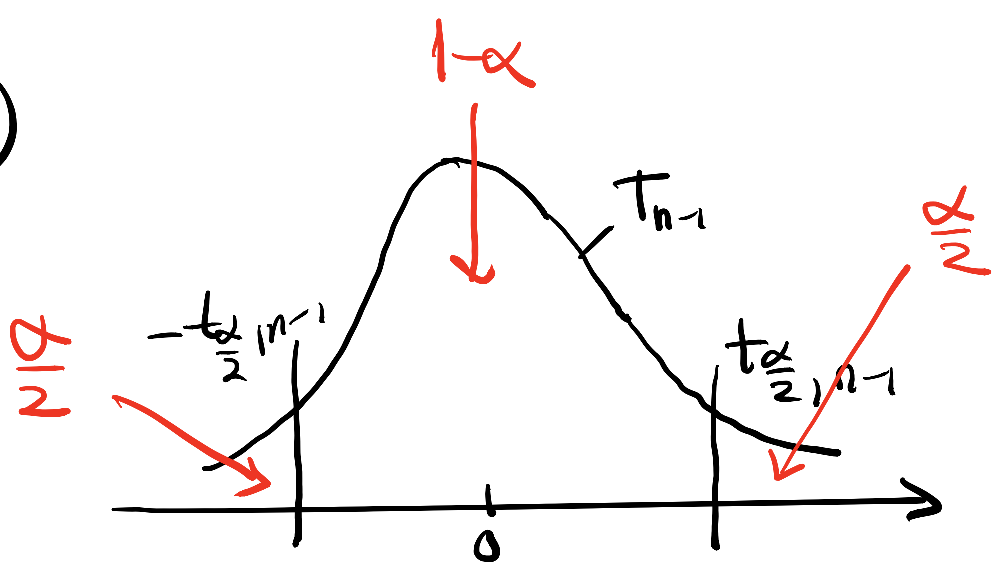
$$Z = \frac{X_0 - \bar{X} - E(X_0 - \bar{X})}{SD(X_0 - \bar{X})} = \frac{X_0 - \bar{X}}{\sigma\sqrt{1+\frac{1}{n}}}$$

$$\sim N(0,1)$$

5)

$$T = \frac{X_0 - \bar{X}}{s\sqrt{1+\frac{1}{n}}} \sim t_{n-1}$$

6)



$$P\left(-t_{\frac{\alpha}{2},n-1} < T < t_{\frac{\alpha}{2},n-1}\right) = 1-\alpha$$

7)

$$\frac{X_0 - \bar{X}}{\sqrt{1+\frac{1}{n}}\, s}$$

Løse ut så X_0 i midten

- $\cdot \left(\sqrt{1+\frac{1}{n}}\, s\right)$
- $+ \bar{X}$

$$P\Big(\underbrace{\bar{X} - t_{\frac{\alpha}{2},n-1}\sqrt{1+\tfrac{1}{n}}\, s}_{X_L} < X_0 < \underbrace{\bar{X} + t_{\frac{\alpha}{2},n-1}\sqrt{1+\tfrac{1}{n}}\, s}_{X_u}\Big) = 1-\alpha$$

Vi setter inn numeriske verdier $(\bar{x}, s, n, t_{\frac{\alpha}{2},n-1})$

og får et $[x_L, x_u]$ $(1-\alpha)100\%$

PI for en fremtidig X_0.

EKS: Høyde kvinner 95% PI

$$\Big[\ \underset{\nearrow\atop 169.5}{\bar{x}} \pm \underbrace{t_{\frac{\alpha}{2},n-1}}_{\substack{t_{0.025,148}\\1.976}} \underbrace{\sqrt{1+\tfrac{1}{n}}}_{\sqrt{1+\frac{1}{149}}}\ \underset{6.6}{s}\ \Big]$$

$= [156.5\ ,\ 182.5]$

Husk 95% KI $[168.9, 170.6]$

$\Rightarrow$ Et PI er mye bredere enn et KI

PI : $\sqrt{1+\frac{1}{n}} \gg \sqrt{\frac{1}{n}}$: KI

To utvalg [9.8, 8.4]

X_{1i} og X_{2j}
uavh. av hverandre

Notasjon:

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X_{1i} (eks: bensinforbruk biltype 1)

eks.

$i = 1, .., n_1$ u.i.f $X_{1i} \sim N(\mu_1, \sigma_1^2 = 2)$

X_{2j} (— " — biltype 2)

eks

$j = 1, .., n_2$ u.i.f $X_{2j} \sim N(\mu_2, \sigma_2^2 = 3)$

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Oppdrag: Estimator & KI for $\mu_1 - \mu_2$

1) Estimator for $\mu_1 - \mu_2$:

$$\bar{X}_1 - \bar{X}_2$$

$$\frac{1}{n_1}\sum_i X_{1i} \qquad \frac{1}{n_2}\sum_j X_{2j}$$

2) Egenskaper:

$\bar{X}_1 - \bar{X}_2$ er normalfordelt fordi differense av to uavh. N-fordelte SV

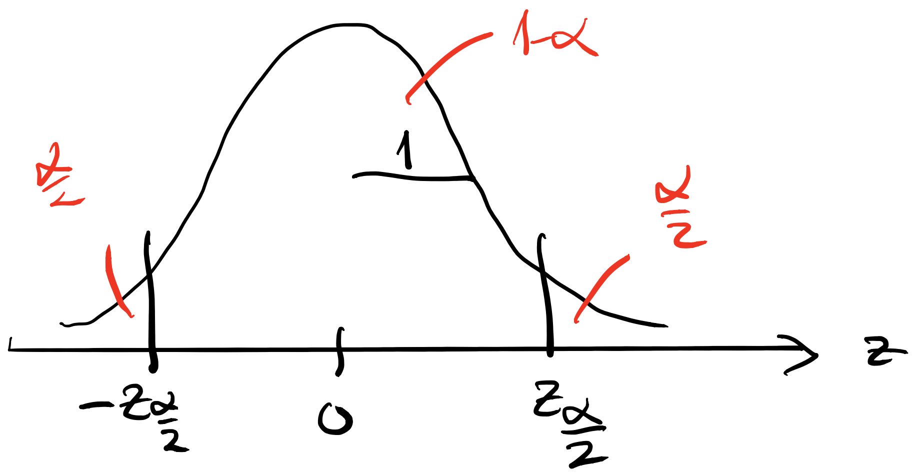
$$E(\bar{X}_1 - \bar{X}_2) = E(\bar{X}_1) - E(\bar{X}_2)$$
$$= \mu_1 - \mu_2$$

$$Var(\bar{X}_1 - \bar{X}_2) \underset{\substack{\uparrow \\ \text{uavhengige}}}{=} Var(\bar{X}_1) + (-1)^2 Var(\bar{X}_2)$$

$$= \underbrace{Var(\bar{X}_1)}_{\frac{\sigma_1^2}{n_1}} + \underbrace{Var(\bar{X}_2)}_{\frac{\sigma_2^2}{n_2}}$$

$$Z = \frac{\bar{X}_1 - \bar{X}_2 - E(\bar{X}_1 - \bar{X}_2)}{SD(\bar{X}_1 - \bar{X}_2)}$$

$$= \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$



$$P\left(-z_{\frac{\alpha}{2}} < Z < z_{\frac{\alpha}{2}}\right) = 1-\alpha$$

$$Z = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

og løs ut så $(\mu_1 - \mu_2)$ i midten

$$(1-\alpha)100\% \text{ KI}: \left[(\bar{x}_1 - \bar{x}_2) \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \right]$$

Hva hvis σ_1 og σ_2 er ulyent?

$$S_1^2 = \frac{1}{n_1-1}\sum_{i=1}^{n_1}(X_{1i}-\bar{X}_1)^2$$

$$S_2^2 = \frac{1}{n_2-1}\sum_{j=1}^{n_2}(X_{2j}-\bar{X}_2)^2$$

$$\frac{\bar{X}_1-\bar{X}_2-(\mu_1-\mu_2)}{\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}} \approx t_\nu$$

funksjon av s_1, s_2, n_1, n_2

KI: $\left[\bar{x}_1-\bar{x}_2 \pm t_{\frac{\alpha}{2},\nu}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}\right]$

ν blir oppgitt på eksamen.

(Innlevering 3: 1c mersvin
→ der må du regne ut ν selv!)