Gamma og Eksponential fordeling

Gamma fordeling

$$f(x;\alpha,\beta)=\begin{cases}\frac{1}{\beta^{\alpha}\Gamma(\alpha)}x^{\alpha-1}e^{-\frac{x}{\beta}}, & x>0,\ \alpha>0,\ \beta>0\\ 0, & \text{elles}\end{cases}$$

$\alpha=1$ gjev eksponentialfordelinga

$$f(x;\beta)=\begin{cases}\frac{1}{\beta}e^{-\frac{x}{\beta}}, & x>0,\ \beta>0\\ 0, & elles\end{cases}$$

$$F_X(x)=\begin{cases}1-e^{-\frac{x}{\beta}}, & x>0\\ 0, & \text{elles}\end{cases}$$

For Poisson prosessen gjev desse fordelinga til hending nummer α

Transformasjonar

Eineintydige $Y=U(X)$, X=W(Y)

$$f_Y(y)=\begin{cases}f_X(W(y)), & \text{diskret fordeling}\\ f_X(W(y))\left|W'(y)\right|, & \text{kontinuerleg fordeling}\end{cases}$$

Momentgenererande funksjonar

$$M_X(t) = \begin{cases} \sum_x e^{tx} f(x), & \text{X diskret} \\ \int_{-\infty}^{\infty} e^{tx} f(x) dx. & \text{X kontinuerleg} \end{cases}$$

$$\frac{d^r M_X(t)}{dt^r}\Big|_{t=0} = E\left[X^r\right]$$

Eigenskapar

$$M_{X+a}(t) = e^{at} M_X(t)$$

$$M_{aX}(t) = M_X(at)$$

$$Y = \sum_{i=1}^{n} X_i \Rightarrow M_Y(t) = \prod_{i=1}^{n} M_{X_i}(t), \quad X_1, X_2, \ldots, X_n \ uavh$$

$$M_Y(t) = M_X(t) \Rightarrow X \ og \ \text{Y har same fordeling}$$

$X_i \sim \text{Poisson}(\nu_i), i = 1, 2, \ldots, n$ og uavhengige. Då er

Y= $\sum_{i=1}^{n} X_i \sim \text{Poisson}\left(\sum_{i=1}^{n} \nu_i\right)$. $\nu_i = \lambda_i t_i$ for poisson prosessen.

Bevis: $X \sim \text{Poisson}(\nu)$

$$\Rightarrow M_X(t) = \sum_{x=0}^{\infty} \frac{e^{tx} \nu^x e^{-\nu}}{x!} = e^{-\nu} \sum_{x=0}^{\infty} \frac{e^{tx} \nu^x}{x!} = e^{-\nu} e^{\nu e^t} = e^{\nu(e^t - 1)}$$

og $M_Y(t) = \prod_{i=1}^{n} e^{\nu_i(e^t-1)} = e^{\sum_{i=1}^{n} \nu_i(e^t - 1)}$.