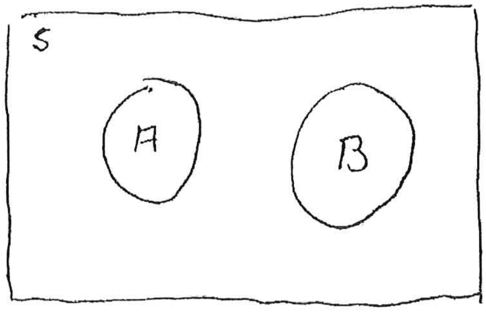
TMA 4240

SANNSYNSREKNING	FORDELINGAR	INFERENS
SANNSYN	STOKASTISKE VARIABLE	SENTRALGRENSE-TEOREMET
ADDISJONSSETNINGA	FORVENTING	ESTIMERING
BETINGA SANNSYN	VARIANS	KONFIDENSINTERVALL
UAVHENGIGHEIT	STANDARDAVVIK	HYPOTESETESTING
MULTIPLIKASJONS-SETNINGA	KOVARIANS	REGRESJON
TOTAL SANNSYN	UNIFORM	
BAYES FORMEL	BINOMISK	
KOMBINATORIKK	GEOMETRISK	
	NEGATIVT BINOMISK	
	HYPERGEOMETRISK	
	POISSON	
	EKSPONENTIAL	
	GAMMA	
	NORMAL	
	χ^2- fordeling	
	t- fordeling	

SANNSYN

STOKASTISKE FORSØK

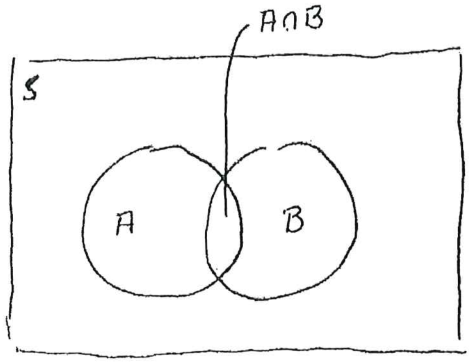


$$P(S) = 1$$

$$0 \leq P(A) \leq 1$$

$$P(A \cup B) = P(A) + P(B), \quad A \cap B = \phi$$

$$P\left(\bigcup_{i=1}^{m} A_i\right) = \sum_{i=1}^{m} P(A_i), \quad A_i \cap A_j = \phi, \ \forall_{ij} \ i \neq j$$



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

BETINGA SANNSYN: $\quad P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

$$P(A \cap B \cap C) = P(C \mid A \cap B)\, P(B|A) \cdot P(A)$$

UAVHENGIGHEIT: $\quad P(A|B) = P(A), \quad P(A|B^c) = P(A)$

$$P(A \cap B) = P(A) \cdot P(B)$$

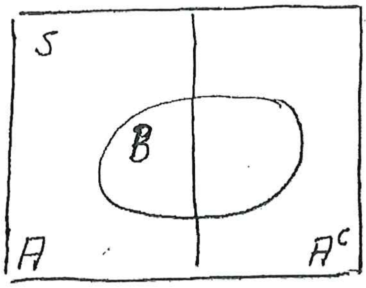
$$P(A_1 \cap \cdots \cap A_m) = P(A_1) \cdot P(A_2) \cdot \ldots \cdot P(A_m)$$

$$P(A_{i_1} \cap A_{i_2} \cap \cdots \cap A_{i_j}) = P(A_{i_1}) \cdot P(A_{i_2}) \cdots P(A_{i_j})$$

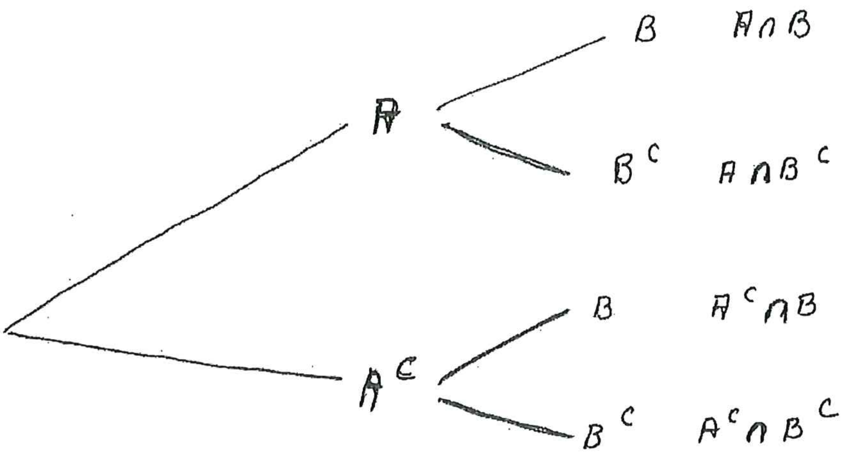
$\forall_j$ og $i_1, \ldots, i_j$

BAYES REGEL

TOTAL SANNSYN



$$P(B) = P(A \cap B) + P(A^c \cap B)$$



$$P(A|B) = \frac{P(A \cap B)}{P(A \cap B) + P(A^c \cap B)}$$

Bayes regel

$$= \frac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P(B|A^c) \cdot P(A^c)}$$

KOMBINATORIKK

UNIFORME SANNSYNSMODELLAR $P(A) = \frac{g}{m}$

MULTIPLIKASJONSREGELEN: $m_1 \cdot m_2 \cdot \cdots \cdot m_k$.

TALET PÅ ORDNA UTVAL $\frac{n!}{(n-s)!} = n(n-1)\cdots(n-s+1)$

TALET PÅ IKKJE ORDNA UTVAL $\frac{n!}{(n-s)!\, s!} = \binom{n}{s}$

FORDELINGSFUNKSJONAR

Diskret

$$F_X(x) = P(X \le x) = \sum_{u \le x} P(X = u)$$

$X = k$ heiltal

$$F_X(k) = P(X \le k) = \sum_{i \le k} P(X = i)$$

$$P(X > k) = P(X \ge k+1) = \sum_{i=k+1}^{\infty} P(X = i)$$

$$P(X = k) = P(X \le k) - P(X \le k-1)$$

Kontinuerlig

$$F_Y(y) = P(Y \le y) = \int_{-\infty}^{y} f_Y(y)\,dy$$

$$f_Y(y) = \frac{d}{dy} F_Y(y)$$

$$P(Y = y) = 0$$

FORVENTING OG VARIANS

$$E[X] = \begin{cases} \int_{-\infty}^{\infty} x\, f(x)\, dx, & X \text{ kontinuerleg} \\ \sum_i x_i\, P(X = x_i), & X \text{ diskret} \end{cases}$$

$$Var[X] = \begin{cases} \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx, & X \text{ kontinuerleg} \\ \sum_i (x_i-\mu)^2\, P(X = x_i), & X \text{ diskret} \end{cases}$$

$$Cov(X,Y) = E[(X-\mu_X)(Y-\mu_Y)]$$

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sigma_X\, \sigma_Y}$$

$$E\left[\sum a_i X_i\right] = \sum a_i E[X_i]$$

$$Var\left[\sum a_i X_i\right] = \sum a_i^2 Var[X_i] + 2 \sum_{i<j}\sum a_i a_j Cov(X_i, X_j)$$

FLEIRDIMENSJONALE FORDELINGSFUNKSJONAR

Diskret

$$F_{X,Y}(x,y) = P(X \le x \cap Y \le y) = \sum_{v \le y} \sum_{u \le x} P(X = u \cap Y = v)$$

$$P(X = x) = \sum_{\text{alle } y} P(X = x \cap Y = y)$$

Kontinuerleg

$$F_{X,Y}(x,y) = \int_{-\infty}^{y} \int_{-\infty}^{x} f_{X,Y}(u,v)\,du\,dv$$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx$$

Uavh.

$$P(X_1 = x_1 \cap X_2 = x_2 \cap \cdots \cap X_n = x_n) = \prod_{i=1}^{n} P(X_i = x_i)$$

$$f_{Y_1,\ldots,Y_n}(y_1,\ldots,y_n) = \prod_{i=1}^{n} f_{Y_i}(y_i)$$

FORVENTING TIL EIN FUNKSJON

$$E[g(X)] = \begin{cases} \sum_{i} g(x_i)\,P(X = x_i), & X \text{ diskret} \\ \int_{-\infty}^{\infty} g(x) f(x)\,dx, & X \text{ kontinuerleg} \end{cases}$$

$$E[g(X,Y)] = \begin{cases} \sum_{y} \sum_{x} g(x,y)\,P(X = x \cap Y = y), & \text{Diskret} \\ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y)\,f_{X,Y}(x,y)\,dx\,dy, & \text{kontinuerleg} \end{cases}$$

MAX OG MIN

$X_1, \ldots X_m$ uavhengige

$P(\max(X_1, \ldots X_m) \leq x) = P(X_1 \leq x \cap X_2 \leq x \cap \cdots \cap X_m \leq x)$

$$= \prod_{i=1}^{m} F_{X_i}(x)$$

$P(\min(X_1, \ldots, X_m) \leq x) = 1 - P(X_1 > x \cap X_2 > x \cap \cdots \cap X_m > x)$

$$= 1 - \prod_{i=1}^{m} P(X_i > x) = \underline{1 - \prod_{i=1}^{m} (1 - F_{X_i}(x))}$$

BETINGA FORDELINGAR

$$P(Y=y \mid X=x) = \frac{P(Y=y \cap X=x)}{P(X=x)}, \quad \text{Diskret}$$

$$f_{Y \mid X=x} = \frac{f_{X,Y}(x,y)}{f_X(x)}, \quad \text{kontinuerleg}$$

$$E[Y \mid X=x] = \begin{cases} \sum_y y\, f(y \mid x), & \text{DISKRET} \\ \int_{-\infty}^{\infty} y\, f(y \mid x)\, dx, & \text{KONTINUERLEG} \end{cases}$$

DISKRETE FORDELINGAR

Binomisk (n, p)

X = talet på gonger A skjer i n forsök

$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$

$E[X] = np, \quad Var[X] = np(1-p)$

Geometrisk (p)

X = talet på forsök til A skjer 1. gong

$P(X=k) = (1-p)^{k-1} p$

$E[X] = \frac{1}{p}, \quad Var[X] = \frac{1-p}{p^2}$

Negativ Binomisk (r, p)

X = talet på forsök til A skjer r-te gong.

$P(X=k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}$

$E[X] = \frac{r}{p}, \quad Var[X] = \frac{r(1-p)}{p^2}$

Hypergeometrisk

X = talet av type A i eit utval av storleik n frå N der r stykker er av type A

$$P(X=k) = \frac{\binom{r}{k}\binom{N-r}{n-k}}{\binom{N}{n}}$$

$E[X] = n\frac{r}{N}, \quad Var[X] = \frac{N-n}{N-1}\, \frac{nr}{N}\left(1-\frac{r}{N}\right)$

TILNÆRMINGAR/RELASJONAR

p liten, $np = \lambda$

$P(X=k) = \frac{(np)^k e^{-np}}{k!}$

$\left.\begin{array}{l} np \geq 5 \\ n(1-p) \geq 5 \end{array}\right\} \Rightarrow X \approx N(np, np(1-p))$

X_i, $i = 1, 2, \dots, r$ uavh. og geometrisk (p)

$\sum_{i=1}^{r} X_i \sim$ Negativ binomisk (r, p)

$\approx N\left(\frac{r}{p}, \frac{r(1-p)}{p^2}\right)$

$N \gg n \Rightarrow$

$P(X=k) \approx \binom{n}{k}\left(\frac{r}{N}\right)^k\left(1-\frac{r}{N}\right)^{n-k}$

$\approx$

Poisson

X = antall hendingar over tid
(på flate, volum o.l.)

$$P(X=x) = e^{-\lambda t}\frac{(\lambda t)^x}{x!}$$

$$E[X] = \lambda t$$

$$Var[X] = \lambda t$$

Kontinuerlege fordelingar

Normal: $f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$

$-\infty < x < \infty$

Eksponential

$$f(x) = \begin{cases} \frac{1}{\beta} e^{-\frac{x}{\beta}}, & x > 0 \\ 0 & \text{elles} \end{cases}$$

$\mu = \beta$, $\sigma^2 = \beta^2$

Gamma

$$f(x) = \begin{cases} \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-\frac{x}{\beta}}, & x > 0 \\ 0 & \text{elles} \end{cases}$$

$\mu = \alpha\beta$, $\sigma^2 = \alpha\beta^2$

Kji-kvadrat

$$f(x) = \begin{cases} \frac{1}{2^{\frac{\nu}{2}}\Gamma(\frac{\nu}{2})} x^{\frac{\nu}{2}-1} e^{-\frac{x}{2}}, & x > 0 \\ 0 & \text{elles} \end{cases}$$

$\nu > 0$, ν heiltal

$\mu = \nu$, $\sigma^2 = 2\nu$

Relasjonar til andre fordelingar

λt stor

$X \sim N(\lambda t, (\sqrt{\lambda t})^2)$

Ventetid mellom to påfølgjande hendingar i Poisson-prosessen

$\beta = \frac{1}{\lambda}$

α heiltal:

Ventetid mellom ei hending og den α neste i Poissonprosessen

$X = \sum_{i=1}^{\nu} U_i^2$, der U_i er $N(0,1)$

og uavh.

Transformasjonsformelen og Moment genererand.

Generelt.

$Y = u(x)$ og eineintydig.

DISKRET : $P(Y=y) = f_X(u^{-1}(y))$

$$F_Y(y) = P(Y \le y) = P(u(X) \le y) = \begin{cases} P(X \le u^{-1}(y)) = F_X(u^{-1}(y)) \\ P(X \ge u^{-1}(y)) = 1 - F_X(u^{-1}(y)) \end{cases}$$

X og Y kontinuerleg fordelte.

$$\Rightarrow \quad g_Y(y) = f_X(u^{-1}(y)) \, | u^{-1}(y)' |$$

$$M_X(t) = E\left[e^{tX}\right] = \begin{cases} \sum_x e^{tx} f(x), & X \text{ diskret} \\ \int_{-\infty}^{\infty} e^{tx} f(x)\,dx, & X \text{ kontinuerleg} \end{cases}$$

$$M_{X+a}(t) = e^{at} M_X(t)$$

$$M_{aX}(t) = M_X(at)$$

$$Y = \sum_{i=1}^{n} X_i, \; X_i \text{ uavh} \Rightarrow M_Y(t) = \prod_{i=1}^{n} M_{X_i}(t)$$

$$\mu_1 = \left. \frac{d\, M_X(t)}{dt} \right|_{t=0} \qquad \mu_2 = \left. \frac{d^2 M_X(t)}{dt^2} \right|_{t=0}.$$

MOMENT GENERERANDE FUNKSJONAR

$$M_X(t) = E[e^{tX}]$$

$$M_{aX+b}(t) = e^{bt} M_X(at)$$

TO RESULTAT

$$E[X^n] = \frac{d^n}{dt^n} M_X(t) \Big|_{t=0}$$

$$M_{\sum_{i=1}^{n} X_i}(t) = \prod_{i=1}^{n} M_{X_i}(t)$$

uavh.

$X_1, X_2, \ldots, X_n$ uavh.

$X_i \sim \text{Poisson}(\lambda_i) \Rightarrow \sum_{i=1}^{n} X_i \sim \text{Poisson}(\sum \lambda_i)$

$X_i \sim B(n_i, p) \Rightarrow \sum_{i=1}^{n} X_i \sim B(\sum_{i=1}^{n} n_i, p)$

$X_i \sim N(\mu_i, \sigma_i^2) \Rightarrow \sum_{i=1}^{n} X_i \sim N(\sum_{i=1}^{n} \mu_i, \sum_{i=1}^{n} \sigma_i^2)$

$X_i \sim$ geometrisk $(p) \Rightarrow \sum_{i=1}^{n} X_i \sim$ Negativ binomisk (n, p)

$X_i \sim$ negativ binomisk $(n_i, p) \Rightarrow \sum_{i=1}^{n} X_i \sim$ Negativ binomisk $(\sum_{i=1}^{n} n_i, p)$

$X_i \sim$ eksponentialfordelt $(\lambda) \Rightarrow \sum_{i=1}^{n} X_i \sim$ gammafordelt (n, λ)

$X_i \sim$ gammafordelt $(n_i, \lambda) \Rightarrow \sum_{i=1}^{n} X_i \sim$ gammafordelt $(\sum_{i=1}^{n} n_i, \lambda)$

SENTRALGRENSETEOREMET

$X_1, \dots, X_n$ tilf. utval $\begin{cases} \text{uavh.} \\ \text{identisk fordelte} \end{cases}$

$E[X_i] = \mu, \qquad Var[X_i] = \sigma^2$

$$E[\bar{X}] = \mu$$

$$Var[\bar{X}] = \frac{\sigma^2}{n}$$

$$\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \xrightarrow[n \to \infty]{} N(0, 1)$$

eller : $\bar{X} \approx N(\mu, \frac{\sigma^2}{n})$

eller : $\sum_{i=1}^{n} X_i \approx N(n\mu, n\sigma^2)$

INFERENS

$X_1, \ldots, X_n$ uavh.

$$L(\theta) = \begin{cases} f_{X_1, \ldots, X_n}(x_1, \ldots, x_n; \theta) = \prod_{i=1}^{n} f_{X_i}(x_i; \theta) \\ P(X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n; \theta) = \prod_{i=1}^{n} P(X_i = x_i; \theta) \end{cases}$$

SSME: θ_e maksimerer $L(\theta)$, evt. $\ln L(\theta)$

$\theta_e = \theta(x_1, \ldots, x_n)$ estimat.

$\hat{\theta} = \hat{\theta}(X_1, \ldots, X_n)$ estimator.

Momentestimatorene: $E[Y^k] = \frac{1}{n} \sum_{i=1}^{n} y_i^k, \quad k = 1, 2, \ldots$

Konfidensintervall

$$P\left(-z_{\frac{\alpha}{2}} \leq \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \leq z_{\frac{\alpha}{2}}\right) = 1 - \alpha$$

$$\Leftrightarrow P\left(\bar{X} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}\right) = 1 - \alpha.$$

$(1-\alpha)100\%$ Konfidensintervall

σ^2 kjend

$$\bar{X} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

Prediksjonsintervall

$$\bar{X} - z_{\frac{\alpha}{2}} \sigma \sqrt{1+\frac{1}{n}} < x_0 < \bar{X} + z_{\frac{\alpha}{2}} \sigma \sqrt{1+\frac{1}{n}}$$

KONFIDENSINTERVALL

$$\bar{X}_1 - \bar{X}_2 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$\hat{p} - z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < p < \hat{p} + z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\hat{p}_1 - \hat{p}_2 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}.$$

σ^2 ukjend

$$\bar{X} - t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}} < \mu < \bar{X} + t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}}$$

Prediksjonsintervall

$$\bar{X} - t_{\frac{\alpha}{2}, n-1}\, s \sqrt{1+\frac{1}{n}} < x_0 < \bar{X} + t_{\frac{\alpha}{2}, n-1}\, s \sqrt{1+\frac{1}{n}}$$

KONFIDENSINTERVALL

$$\bar{X}_1 - \bar{X}_2 \pm t_{\frac{\alpha}{2}, n_1+n_2-2}\, s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}.$$

$$\bar{X}_1 - \bar{X}_2 \pm t_{\frac{\alpha}{2}, \nu} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}.$$

$$\frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2}}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2}}}.$$

BEGREP I HYPOTESETESTING

$\alpha =$ P(type 1 feil) = signifikansnivå

= P(forkaste H_0 | H_0 er sann)

$\beta =$ P(type 2 feil) = P(ikkje forkaste H_0 | H_0 er gal)

$1-\beta =$ P(forkaste H_0 | H_0 er gal) blir kalla styrken av ein test.

t- fordeling

$$T = \frac{N(0,1^2)}{\sqrt{\frac{\chi^2(\nu)}{\nu}}} \sim t_\nu$$

Eks.

$$\frac{\frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}}}{\sqrt{\frac{S^2(n-1)}{\sigma^2(n-1)}}} = \frac{\bar{X}-\mu}{\frac{S}{\sqrt{n}}} \sim t_{n-1}$$

P-verdi = sannsynet for å få noko som er minst like ekstremt som det som er observert gitt at H_0 er rett.

TABLE 10.2 Tests Concerning Means

H_0	Value of test statistic	H_1	Critical region
$\mu = \mu_0$	$z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$; σ known	$\mu < \mu_0$ $\mu > \mu_0$ $\mu \neq \mu_0$	$z < -z_\alpha$ $z > z_\alpha$ $z < -z_{\alpha/2}$ and $z > z_{\alpha/2}$
$\mu = \mu_0$	$t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$; $v = n - 1$, σ unknown	$\mu < \mu_0$ $\mu > \mu_0$ $\mu \neq \mu_0$	$t < -t_\alpha$ $t > t_\alpha$ $t < -t_{\alpha/2}$ and $t > t_{\alpha/2}$
$\mu_1 - \mu_2 = d_0$	$z = \dfrac{(\bar{x}_1 - \bar{x}_2) - d_0}{\sqrt{(\sigma_1^2/n_1) + (\sigma_2^2/n_2)}}$; σ_1 and σ_2 known	$\mu_1 - \mu_2 < d_0$ $\mu_1 - \mu_2 > d_0$ $\mu_1 - \mu_2 \neq d_0$	$z < -z_\alpha$ $z > z_\alpha$ $z < -z_{\alpha/2}$ and $z > z_{\alpha/2}$
$\mu_1 - \mu_2 = d_0$	$t = \dfrac{(\bar{x}_1 - \bar{x}_2) - d_0}{s_p\sqrt{(1/n_1) + (1/n_2)}}$; $v = n_1 + n_2 - 2$, $\sigma_1 = \sigma_2$ but unknown $s_p^2 = \dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$	$\mu_1 - \mu_2 < d_0$ $\mu_1 - \mu_2 > d_0$ $\mu_1 - \mu_2 \neq d_0$	$t < -t_\alpha$ $t > t_\alpha$ $t < -t_{\alpha/2}$ and $t > t_{\alpha/2}$
$\mu_1 - \mu_2 = d_0$	$t' = \dfrac{(\bar{x}_1 - \bar{x}_2) - d_0}{\sqrt{(s_1^2/n_1) + (s_2^2/n_2)}}$; $v = \dfrac{(s_1^2/n_1 + s_2^2/n_2)^2}{\dfrac{(s_1^2/n_1)^2}{n_1 - 1} + \dfrac{(s_2^2/n_2)^2}{n_2 - 1}}$; $\sigma_1 \neq \sigma_2$ and unknown	$\mu_1 - \mu_2 < d_0$ $\mu_1 - \mu_2 > d_0$ $\mu_1 - \mu_2 \neq d_0$	$t' < -t_\alpha$ $t' > t_\alpha$ $t' < -t_{\alpha/2}$ and $t' > t_{\alpha/2}$
$\mu_D = d_0$ paired observations	$t = \dfrac{\bar{d} - d_0}{s_d/\sqrt{n}}$; $v = n - 1$, $\mu_D > d_0$	$\mu_D < d_0$ $t > t_\alpha$ $\mu_D \neq d_0$	$t < -t_\alpha$ $t < -t_{\alpha/2}$ and $t > t_{\alpha/2}$

is the standard normal variable Z. Therefore,

$$\beta = P\left(Z < \frac{a - \mu_0}{\sigma/\sqrt{n}} - \frac{\delta}{\sigma/\sqrt{n}}\right) = P\left(Z < z_\alpha - \frac{d}{\sigma/\sqrt{n}}\right),$$

from which we conclude that

$$-z_\beta = z_\alpha - \frac{\delta\sqrt{n}}{\sigma}$$

Kva skal de kunne om Regresjonsanalyse

Enkel lineær regresjon (lineær i koeffisientane)

$$Y_i = \alpha + \beta x_i + \varepsilon_i \sim \begin{cases} N(0, \sigma^2) \\ \text{og uavh.} \end{cases}$$

Finn a og b ved å minimere

$$Q = \sum_i (y_i - a - bx_i)^2 \quad \text{m.o.p. a og b}$$

$$B = \frac{\sum (x_i - \bar{x}) Y_i}{\sum (x_i - \bar{x})^2}$$

$$A = \bar{Y} - B\bar{x}$$

KVA BRUKAR EIN AV TIDLEGARE LÆRT

KUNNSKAP?

$$E[\hat{\beta}_1] = \beta_1, \qquad E[\hat{\beta}_0] = \beta_0$$

FORVENTNING TIL LINEÆKOMBINASJONAR AV TILFELDIGE VARIABLE

$$Var[\hat{\beta}_1] = \frac{\sigma^2}{\sum(x_i-\bar{x})^2}$$

$$Var[\hat{\beta}_0] = \frac{\sigma^2 \sum_i^n x_i^2}{n\sum(x_i-\bar{x})^2}$$

Varians til lineær kombinasjonar av uavhengige tilfeldige variable.

$$\hat{\beta}_1 \sim N\left(\beta_1, \frac{\sigma^2}{\sum(x_i-\bar{x})^2}\right)$$

$$\hat{\beta}_0 \sim N\left(\beta_0, \frac{\sigma^2 \sum x_i^2}{n\sum(x_i-\bar{x})^2}\right)$$

Fordeling til lineærkombinasjonar av uavh. normalfordelte variable

Konfidensintervall for β_0 og β_1

Konfidensintervall med normalfordeling og t-fordeling.

$$b_0 \pm z_{\frac{\alpha}{2}} \cdot \sigma \sqrt{\frac{\sum x_i^2}{n\sum(x_i-\bar{x})^2}}.$$

$$b_1 \pm t_{\frac{\alpha}{2}, n-2} \cdot s \sqrt{\frac{1}{\sum(x_i-\bar{x})^2}}.$$

HYPOTESTESTING

$H_0: \beta_1 = \beta_{10}$ $\quad H_1: \beta_1 \overset{>}{\underset{<}{\neq}} \beta_{10}$

$H_0: \beta_0 = \beta_{00}$ $\quad H_1: \beta_0 \overset{>}{\underset{<}{\neq}} \beta_{00}$

Hypotesetesting i normalfordeling og t-fordeling.

Konfidensintervall for

$E[Y \mid x = x_0]$, μ_{Y_0}

$\quad \shortparallel$

$\beta_0 + \beta_1 x_0$

Konfidensintervall for normalfordeling og t-fordeling.

$$\hat{\mu}_{Y_0} = \hat{\beta}_0 + \hat{\beta}_1 x_0 \sim N\left(\beta_0 + \beta_1 x_0,\ \sigma^2\left(\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}\right)\right)$$

Prediksjonsintervall for

Y_0 i $x = x_0$

$\hat{Y}_0 = \hat{\beta}_0 + \hat{\beta}_1 x_0$.

Prediksjonsintervall med normalfordeling og t-fordeling.

$$Y_0 - \tilde{Y}_0 \sim N\left(0,\ \sigma^2\left(1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}\right)\right)$$

EKSAMENSRÅD

- Løys minst 3 eksamenssett.
- Gjer dykk kjend med „Tabeller og formler..." og all informasjon som finst der.
- Legg arbeid i det gule arket.
- Ver ikkje redd for å hoppe over punkt for evt. å ta dei seinare.
- Om de synest eit punkt er vanskeleg, sjå om de kan svare på delpunkt.
- Evaluer svara. NB! Aldri sannsyn > 1
  Aldri negativ varians.
- Tru ikkje at vi går inn for å lure dykk.