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Department of Mathematical Sciences

Examination paper for TMA4240/TMA4245 Statistics

Examination date: Friday, August 13th, 2021

Examination time (from-to): 09.00 – 13.00

Permitted examination support material: A / All support material is allowed

Academic contact during examination: Håkon Tjelmeland
Phone: 4822

Technical support during examination: Orakel support services
Phone: 73 59 16 00

If you experience technical problems during the exam, contact Orakel support services as soon as possible before the examination time expires. If you don't get through immediately, hold the line until your call is answered.

OTHER INFORMATION

Make your own assumptions: If a question is unclear/vague, make your own assumptions and specify them in your answer. Only contact academic contact in case of errors or insufficiencies in the question set.

Cheating/Plagiarism: The exam is an individual, independent work. Examination aids are permitted, but make sure you follow any instructions regarding citations. During the exam it is not permitted to communicate with others about the exam questions, or distribute drafts for solutions. Such communication is regarded as cheating. All submitted answers will be subject to plagiarism control. *Read more about cheating and plagiarism here.*

Notifications: If there is a need to send a message to the candidates during the exam (e.g. if there is an error in the question set), this will be done by sending a notification in Inspera. A dialogue box will appear. You can re-read the notification by clicking the bell icon in the top right-hand corner of the screen. All candidates will also receive an SMS to ensure that nobody misses out on important information. Please keep your phone available during the exam.

Weighting: The weighting of the problems is specified in each problem.

General instructions: In Problems 1 to 5 you should only give the correct answer without any justification. Use more significant digits during calculations than in the final answers to avoid errors due to rounding. **Follow the instructions regarding decimal places in the final answers. If not, the answer may be evaluated as wrong.** For Problems 6 to 11 all answers must be justified and all natural intermediate calculations must be included. It must be completely clear how you arrived at the final answers. The solutions to these problems should be hand written.

ABOUT SUBMISSION

How to answer questions: All question types other than Upload assignment must be answered directly in Inspera. In Inspera, your answers are saved automatically every 15 seconds. **NB!** We advise against pasting content from other programs, as this may cause loss of formatting and/or entire elements (e.g. images, tables).

File upload: All files must be uploaded before the examination time expires. All uploaded files must be in "pdf"-format. 30 minutes are added to the examination time to manage the sketches/calculations/files. The additional time is included in the remaining examination time shown in the top left-hand corner.

NB! You are responsible to ensure that the file(s) are correct and not corrupt/damaged. Check the file(s) you have uploaded by clicking “Download” when viewing the question. All files can be removed or replaced as long as the test is open.

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Automatic submission: Your answer will be submitted automatically when the examination time expires and the test closes, if you have answered at least one question. This will happen even if you do not click “Submit and return to dashboard” on the last page of the question set. You can reopen and edit your answer as long as the test is open. If no questions are answered by the time the examination time expires, your answer will not be submitted. This is considered as “did not attend the exam”.

Withdrawing from the exam: If you become ill, or wish to submit a blank test/withdraw from the exam for another reason, go to the menu in the top right-hand corner and click “Submit blank”. This cannot be undone, even if the test is still open.

Accessing your answer post-submission: You will find your answer in Archive when the examination time has expired.

1 **1A**

Introduction: We observe a random sample consisting of the values -1, 2, 3, 7, 9.

Exercise: Compute the following. Enter the answers with two decimal places.

- Sample mean:
- Sample variance (empirical variance):
- Median:

Maximum marks: 5

2 **1B**

Introduction : We observe a random sample consisting of the values 9, 12, 13, 17, 19.

Exercise: Compute the following. Enter the answers with two decimal places.

- Sample mean:
- Sample variance (empirical variance):
- Median:

Maximum marks: 5

3 1C

Introduction: We observe a random sample consisting of the values -1, 2, 10, 11, 13.

Exercise: Compute the following. Enter the answers with two decimal places.

- Sample mean:
- Sample variance (empirical variance):
- Median:

Maximum marks: 5

4 1D

Introduction: We observe a random sample consisting of the values 0, 4, 5, 8, 8.

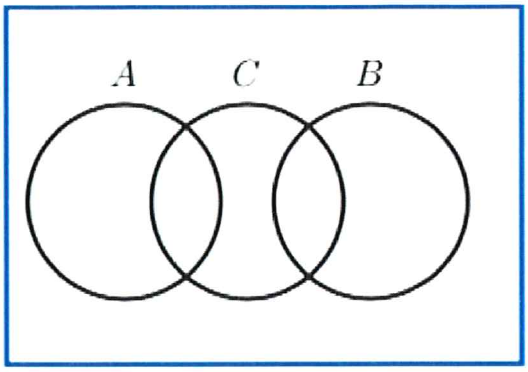
Exercise: Compute the following. Enter the answers with two decimal places.

- Sample mean:
- Sample variance (empirical variance):
- Median:

Maximum marks: 5

2A

Introduction: Let A, B and C be three events in the sample space S, where events A and B are disjoint. The events can be illustrated with a Venn diagram as follows:



In addition, let the following probabilities be given as:

- $P(A) = 0.08$
- $P(B) = 0.20$
- $P(C|A) = 0.25$
- $P(C|B) = 0.50$
- $P(A|C) = 0.10$

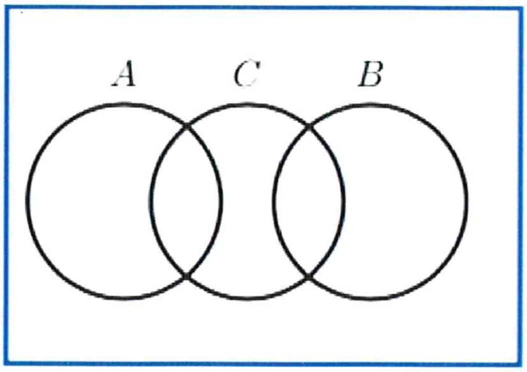
Exercise : Find the following probabilities. Give the answers with three decimal positions.

- $P(B \cap C) =$
- $P(C) =$
- $P(B|A \cup C) =$

Maximum marks: 5

6 **2B**

Introduction : Let A, B and C be three events in the sample space S, where events A and B are disjoint. The events can be illustrated with a Venn diagram as follows:



In addition, let the following probabilities be given as:

- $P(A) = 0.40$
- $P(B) = 0.20$
- $P(C|A) = 0.25$
- $P(C|B) = 0.20$
- $P(A|C) = 0.20$

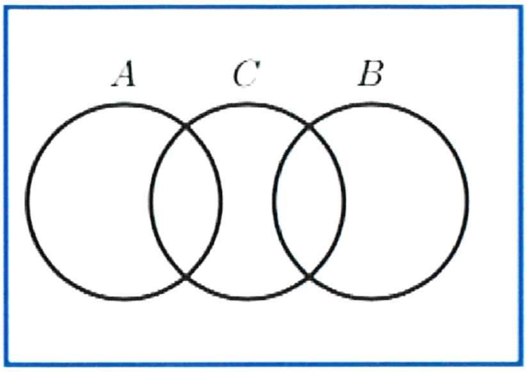
Exercise : Find the following probabilities. Give the answers with three decimal positions.

- $P(B \cap C) =$
- $P(C) =$
- $P(B|A \cup C) =$

Maximum marks: 5

2C

Introduction: Let A, B and C be three events in the sample space S, where events A and B are disjoint. The events can be illustrated with a Venn diagram as follows:



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In addition, let the following probabilities be given as:

- $P(A) = 0.10$
- $P(B) = 0.25$
- $P(C|A) = 0.90$
- $P(C|B) = 0.10$
- $P(B|C) = 0.20$

Exercise : Find the following probabilities. Give the answers with three decimal positions.

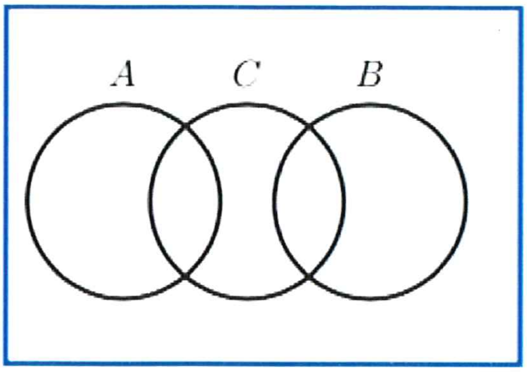
- $P(A \cap C) =$ ☐
- $P(C) =$ ☐
- $P(A|B \cup C) =$ ☐

Maximum marks: 5

8

2D

Introduction : Let A, B and C be three events in the sample space S, where events A and B are disjoint. The events can be illustrated with a Venn diagram as follows:



.

In addition, let the following probabilities be given as:

- $P(A) = 0.50$
- $P(B) = 0.10$
- $P(C|A) = 0.20$
- $P(C|B) = 0.05$
- $P(B|C) = 0.10$

Exercise : Find the following probabilities. Give the answers with three decimal positions.

- $P(A \cap C) =$
- $P(C) =$
- $P(A|B \cup C) =$

Maximum marks: 5

9 **3A**

Introduction : Assume that we repeatedly draw a ball from an urn with 4 red and 16 blue balls. Assume moreover that each time we put the ball back in the urn before drawing again. After each draw, we record the color of the ball before putting it back again.

Exercise: When we consider each ball as unique, in how many ways can we:

- draw 5 balls so that you draw a red ball for the first time on draw number 5? Enter the answer as an integer.
- draw 7 balls so that you draw a red ball for the fourth time on draw number 7? Enter the answer as an integer.

Maximum marks: 5

10 **3B**

Introduction : Assume that we repeatedly draw a ball from an urn with 5 red and 13 blue balls. Assume moreover that each time we put the ball back in the urn before drawing again. After each draw, we record the color of the ball before putting it back again.

Exercise: When we consider each ball as unique, in how many ways can we:

- draw 6 balls so that you draw a red ball for the first time on draw number 6? Enter the answer as an integer.
- draw 8 balls so that you draw a red ball for the third time on draw number 8? Enter the answer as an integer

Maximum marks: 5

11 3C

Introduction : Assume that we repeatedly draw a ball from an urn with 6 red and 16 blue balls. Assume moreover that each time we put the ball back in the urn before drawing again. After each draw, we record the color of the ball before putting it back again.

Exercise: When we consider each ball as unique, in how many ways can we:

- draw 4 balls so that you draw a red ball for the first time on draw number 4? Enter the answer as an integer.
- draw 6 balls so that you draw a red ball for the fifth time on draw number 6? Enter the answer as an integer.

Maximum marks: 5

12 4A

Introduction : Let X be a discrete stochastic variable with probability mass function $f(x)$ given in the following table:

x	0	1	2	3	4	5
$f(x)$	0.05	0.15	0.10	0.25	0.20	0.25

Exercise: Find the following quantities. Enter the answers with three decimal positions.

- $P(X > 1) =$
- $P(X \leq 4 | X \geq 3) =$
- $\mathrm{E}[X] =$

Maximum marks: 5

13 4B

Introduction : Let X be a discrete stochastic variable with probability mass function $f(x)$ given in the following table:

x	0	1	2	3	4	5
$f(x)$	0.10	0.15	0.20	0.25	0.15	0.15

Exercise: Find the following quantities. Enter the answers with three decimal positions.

- $P(X > 1) =$
- $P(X \leq 4 | X \geq 3) =$
- $\mathrm{E}[X] =$

Maximum marks: 5

14 4C

Introduction : Let X be a discrete stochastic variable with probability mass function $f(x)$ given in the following table:

x	0	1	2	3	4	5
$f(x)$	0.05	0.15	0.10	0.25	0.20	0.25

Exercise: Find the following quantities. Enter the answers with three decimal positions.

- $P(X > 2) =$ ☐
- $P(X < 5 | X \geq 1) =$ ☐
- $\mathrm{E}[X] =$ ☐

Maximum marks: 5

15 **4D**

Introduction : Let X be a discrete stochastic variable with probability mass function $f(x)$ given in the following table:

x	0	1	2	3	4	5
$f(x)$	0.10	0.15	0.20	0.25	0.15	0.15

Exercise: Find the following quantities. Enter the answers with three decimal positions.

- $P(X > 2) =$
- $P(X < 5 | X \geq 1) =$
- $\mathrm{E}[X] =$

Maximum marks: 5

16 **5A**

Introduction : Assume that X is exponentially distributed and that $\mathrm{Var}(X) = 1/9$.

Exercise: Compute the following quantities. Enter the answers with three decimal positions.

- $E(X)$:
- $P(X > 1)$:
- $P(X \leq 0.5 | X < 1.1)$:

Maximum marks: 5

17 5B

Introduction: Assume that X is exponentially distributed and that $\mathrm{Var}(X) = 1/16$.

Exercise: Compute the following quantities. Enter the answers with three decimal positions.

- $E(X)$:
- $P(X > 1)$:
- $P(X \leq 0.5 | X < 1.1)$:

Maximum marks: 5

18 5C

Introduction: Assume that X is exponentially distributed and that $\mathrm{Var}(X) = 1/4$.

Exercise: Compute the following quantities. Enter the answers with three decimal positions.

- $E(X)$:
- $P(X > 1)$:
- $P(X \leq 0.5 | X < 1.1)$:

Maximum marks: 5

19 **6A**

Introduction : Let X and Y be independent and normally distributed stochastic variables, where X has mean value μ and variance σ^2, while Y has mean value 2μ and variance $4\sigma^2$.

Assume that the value of the parameter μ is unknown and that we wish to estimate it based on X and Y. The following three estimators for μ are proposed

$$\widehat{\mu} = \frac{X+Y}{2}, \qquad \mu^{\star} = \frac{2X+Y}{4} \qquad \text{and} \qquad \widetilde{\mu} = \frac{3X+Y}{5}.$$

Exercise: Which of the the three proposed estimators $\widehat{\mu}$, $\mu^{\star}$ and $\widetilde{\mu}$ would you prefer? Justify your answer.

Note: You must upload a pdf file here that contains image(s) of your **handwritten** solution to the problem. When correcting this assignment, emphasis will be placed on the answer being **logical** and containing **all natural intermediate calculations.**

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Maximum marks: 10

20 **6B**

Introduction: Let X and Y e independent and normally distributed stochastic variables, where X has mean value μ and variance σ^2, while Y had mean value 2μ and variance $4\sigma^2$.

Assume that the value of the parameter μ is unknown and that we wish to estimate it based on X and Y. The following three estimators for μ are proposed

$$\widehat{\mu} = \frac{2X+Y}{4}, \qquad \mu^{\star} = \frac{3X+Y}{5} \qquad \text{og} \qquad \widetilde{\mu} = \frac{X+Y}{2}.$$

Exercise: Which of the the three proposed estimators $\widehat{\mu}$, $\mu^{\star}$ and $\widetilde{\mu}$ would you prefer? Justify your answer.

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Maximum marks: 10

21 **6C**

Introduction: Let X and Y be independent and normally distributed stochastic variables, where X has mean value μ and variance σ^2, while Y has mean value 2μ and variance $4\sigma^2$.

Assume that the value of the parameter μ is unknown and that we wish to estimate it based on X and Y. The following three estimators for μ are proposed

$$\widehat{\mu} = \frac{3X+Y}{5}, \qquad \mu^{\star} = \frac{X+Y}{2} \qquad \text{og} \qquad \widetilde{\mu} = \frac{2X+Y}{4}.$$

Exercise: Which of the the three proposed estimators $\widehat{\mu}$, $\mu^{\star}$ and $\widetilde{\mu}$ would you prefer? Justify your answer.

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Maximum marks: 10

7A

Introduction: Let X and Y be continuous stochastic variables with joint probability density

$$f(x,y) = \begin{cases} \frac{1}{4}\exp\{-\left[x + \frac{1}{2}|y-x|\right]\} & \text{for } x > 0,\ -\infty < y < \infty, \\ 0 & \text{otherwise.} \end{cases}$$

Exercise: Find the following distributions:

- The marginal distribution of X.
- The conditional distribution of Y given that $X = x$.

Note: You must upload a pdf file here that contains image(s) of your **handwritten** solution to the problem. When correcting this assignment, emphasis will be placed on the answer being **logical** and containing **all natural intermediate calculations.**

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All file types are allowed. Maximum file size is **50 GB**

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Maximum marks: 10

23 **7B**

Introduction: Let X and Y be continuous stochastic variables with joint probability density

$$f(x,y) = \begin{cases} \frac{1}{4}\exp\{-\left[\frac{1}{2}x + |y-x|\right]\} & \text{for } x > 0, -\infty < y < \infty, \\ 0 & \text{otherwise.} \end{cases}$$

Exercise: Find the following distributions:

- The marginal distribution of X.
- The conditional distribution of Y given that $X = x$.

Note: You must upload a pdf file here that contains image(s) of your **handwritten** solution to the problem. When correcting this assignment, emphasis will be placed on the answer being **logical** and containing **all natural intermediate calculations.**

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All file types are allowed. Maximum file size is **50 GB**

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Maximum marks: 10

24 **7C**

Introduction: Let X and Y be continuous stochastic variables with joint probability density

$$f(x,y) = \begin{cases} 2\exp\{-2\,[x + |y - x|]\} & \text{for } x > 0, -\infty < y < \infty, \\ 0 & \text{otherwise.} \end{cases}$$

Exercise: Find the following distributions:

- The marginal distribution of X.
- The conditional distribution of Y given that $X = x$.

Note: You must upload a pdf file here that contains image(s) of your **handwritten** solution to the problem. When correcting this assignment, emphasis will be placed on the answer being **logical** and containing **all natural intermediate calculations.**

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Maximum marks: 10

25 **8A**

Introduction : Assume that we observe a random sample consisting of $n = 6$ observations 1, 5, 2, 10, 15, 3 from a geometric distribution with probability mass function

$$f(x) = (1-p)^{x-1}p \text{ for } x = 1, 2, \ldots,$$

where $p \in (0, 1)$ is a parameter.

Exercise:

- Find the maximum likelihood estimator for *p* and compute the estimate for the data given above.
- Find also the maximum likelihood estimator $\hat{\theta}$ for $\theta = 1/p$.
- Find out if $\hat{\theta}$ is an unbiased estimator for θ.

Note: You must upload a pdf file here that contains image(s) of your **handwritten** solution to the problem. When correcting this assignment, emphasis will be placed on the answer being **logical** and containing **all natural intermediate calculations.**

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Maximum marks: 10

26 **8B**

Introduction : Assume that we observe a random sample consisting of $n = 5$ observations 5, 15, 13, 4, 17 from a geometric distribution with probability mass function

$$f(x) = (1-p)^{x-1}p \text{ for } x = 1, 2, \ldots,$$

where $p \in (0, 1)$ is a parameter.

Exercise:

- Find the maximum likelihood estimator for p and compute the estimate for the data given above.
- Find also the maximum likelihood estimator $\hat{\theta}$ for $\theta = 1/p$.
- Find out if $\hat{\theta}$ is an unbiased estimator for θ.

Note: You must upload a pdf file here that contains image(s) of your **handwritten** solution to the problem. When correcting this assignment, emphasis will be placed on the answer being **logical** and containing **all natural intermediate calculations.**

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Maximum marks: 10

27 **8C**

Introduction : Assume that we observe a random sample consisting of $n = 7$ observations 2, 5, 4, 4, 1, 9, 5 from a geometric distribution with probability mass function

$$f(x) = (1-p)^{x-1}p \text{ for } x = 1, 2, \ldots,$$

where $p \in (0, 1)$ is a parameter.

Exercise:

- Find the maximum likelihood estimator for p and compute the estimate for the data given above.
- Find also the maximum likelihood estimator $\hat{\theta}$ for $\theta = 1/p$.
- Find out if $\hat{\theta}$ is an unbiased estimator for θ.

Note: You must upload a pdf file here that contains image(s) of your **handwritten** solution to the problem. When correcting this assignment, emphasis will be placed on the answer being **logical** and containing **all natural intermediate calculations.**

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Maximum marks: 10

28 9A

Introduction : Let X be a continuous stochastic variable with probability density

$$f(x) = \begin{cases} \frac{1}{12}(x+3) & \text{for } x \in (-2, 2), \\ 0 & \text{otherwise.} \end{cases}$$

Exercise: Find the probability $P\left(X^2 - \frac{9X}{4} > -\frac{7}{8}\right)$. *Hint: Find first for which values of* X *the inequality is fulfilled.*

Note: You must upload a pdf file here that contains image(s) of your **handwritten** solution to the problem. When correcting this assignment, emphasis will be placed on the answer being **logical** and containing **all natural intermediate calculations.**

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Maximum marks: 10

29 9B

Introduction : Let X be a continuous stochastic variable with probability density

$$f(x) = \begin{cases} \frac{1}{50}(x+5) & \text{for } x \in (-5,5), \\ 0 & \text{otherwise.} \end{cases}$$

Exercise: Find the probability $P\left(X^2 - \frac{9X}{4} > -\frac{7}{8}\right)$. *Hint: Find first for which values of* X *the inequality is fulfilled.*

Note: You must upload a pdf file here that contains image(s) of your **handwritten** solution to the problem. When correcting this assignment, emphasis will be placed on the answer being **logical** and containing **all natural intermediate calculations.**

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Maximum marks: 10

30 **9C**

Introduction : Let X be a continuous stochastic variable with probability density

$$f(x) = \begin{cases} \frac{1}{12}(x+3) & \text{for } x \in (-2,2), \\ 0 & \text{otherwise.} \end{cases}$$

Exercise: Find the probability $P\left(X^2 - 3X > -\frac{5}{4}\right)$. *Hint: Find first for which values of X the inequality is fulfilled.*

Note: You must upload a pdf file here that contains image(s) of your **handwritten** solution to the problem. When correcting this assignment, emphasis will be placed on the answer being **logical** and containing **all natural intermediate calculations.**

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Maximum marks: 10

31 **9D**

Introduction : Let X be a continuous stochastic variable with probability density

$$f(x) = \begin{cases} \frac{1}{50}(x+5) & \text{for } x \in (-5,5), \\ 0 & \text{otherwise.} \end{cases}$$

Exercise: Find the probability $P\left(X^2 - 3X > -\frac{5}{4}\right)$. *Hint: Find first for which values of* X *the inequality is fulfilled.*

Note: You must upload a pdf file here that contains image(s) of your **handwritten** solution to the problem. When correcting this assignment, emphasis will be placed on the answer being **logical** and containing **all natural intermediate calculations.**

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Maximum marks: 10

32 **10A**

Introduction: An established vaccine confers immunity against a known viral infection among 90% of those vaccinated. A pharmaceutical company has developed a new vaccine and they want to investigate whether the new vaccine is more effective than the vaccine already in use, i.e. whether the proportion of the population that becomes immune when using the new vaccine is higher than among those who receive the old one.

They are investigating this by examining 1000 subjects vaccinated with the new vaccine for the presence of antibodies against the virus. Assume that the pharmaceutical company finds antibodies in $x = 912$ of the subjects and that we can thus conclude that they have acquired immunity.

Exercise:

a)

- Formulate and justify a statistical model for the data in the experiment.
- Formulate the null and alternative hypotheses in the situation described above.
- Find a test statistic and explain why its distribution under the null hypotheses can be approximated with a normal distribution.
- What is the rejection region if we use a significance level of 5%. What is the conclusion from the test?

b)

- Assume that the new vaccine, in reality, confers immunity to 92% of those vaccinated. Derive an approximated expression for the probability that we will reject the null hypotheses above. Determine a numerical value for this probability.
- How many persons must be included in the trial for the test power to be at least 80%, given that the new vaccine confers immunity to 92% of those vaccinated.

Note: You must upload a pdf file here that contains image(s) of your **handwritten** solution to the problem. When correcting this assignment, emphasis will be placed on the answer being **logical** and containing **all natural intermediate calculations.**

Upload your file here. Maximum one file.

All file types are allowed. Maximum file size is **50 GB**

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Maximum marks: 20

33 10B

Introduction: An established vaccine confers immunity against a known viral infection among 80% of those vaccinated. A pharmaceutical company has developed a new vaccine and they want to investigate whether the new vaccine is more effective than the vaccine already in use, i.e. whether the proportion of the population that becomes immune when using the new vaccine is higher than among those who receive the old one.

They are investigating this by examining 1000 subjects vaccinated with the new vaccine for the presence of antibodies against the virus. Assume that the pharmaceutical company finds antibodies in $x = 815$ of the subjects and that we can thus conclude that they have acquired immunity.

Oppgave:

a)

- Formulate and justify a statistical model for the data in the experiment.
- Formulate the null and alternative hypotheses in the situation described above.
- Find a test statistic and explain why its distribution under the null hypotheses can be approximated with a normal distribution.
- What is the rejection region if we use a significance level of 1%. What is the conclusion from the test?

b)

- Assume that the new vaccine, in reality, confers immunity to 83% of those vaccinated. Derive an approximated expression for the probability that we will reject the null hypotheses above. Determine a numerical value for this probability.
- How many persons must be included in the trial for the test power to be at least 80%, given that the new vaccine confers immunity to 83% of those vaccinated.

Note: You must upload a pdf file here that contains image(s) of your **handwritten** solution to the problem. When correcting this assignment, emphasis will be placed on the answer being **logical** and containing **all natural intermediate calculations.**

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All file types are allowed. Maximum file size is **50 GB**

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Maximum marks: 20

34 10C

Introduction: An established vaccine confers immunity against a known viral infection among 95% of those vaccinated. A pharmaceutical company has developed a new vaccine and they want to investigate whether the new vaccine is more effective than the vaccine already in use, i.e. whether the proportion of the population that becomes immune when using the new vaccine is higher than among those who receive the old one.

They are investigating this by examining 1000 subjects vaccinated with the new vaccine for the presence of antibodies against the virus. Assume that the pharmaceutical company finds antibodies in $x = 961$ of the subjects and that we can thus conclude that they have acquired immunity.

Exercise:

a)

- Formulate and justify a statistical model for the data in the experiment.
- Formulate the null and alternative hypotheses in the situation described above.
- Find a test statistics and explain why its distribution under the null hypotheses can be approximated with a normal distribution.
- What is the rejection region if we use a significance level of 5%. What is the conclusion from the test?

b)

- Assume that the new vaccine, in reality, confers immunity to 96% of those vaccinated. Derive an approximated expression for the probability that we will reject the null hypotheses above. Determine a numerical value for this probability.
- How many persons must be included in the trial for the test power to be at least 80%, given that the new vaccine confers immunity to 96% of those vaccinated.

Note: You must upload a pdf file here that contains image(s) of your **handwritten** solution to the problem. When correcting this assignment, emphasis will be placed on the answer being **logical** and containing **all natural intermediate calculations.**

Upload your file here. Maximum one file.

All file types are allowed. Maximum file size is **50 GB**

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Maximum marks: 20

35 **11A**

Intoduction: Assume that we observe a random sample consisting of $n = 2$ observations X_1, X_2 from a normal distribution with unknown mean and variance.

Exercise:

- Derive a $(1 - \alpha)$-prediction interval for a new observation X_3.
- Compute the interval given that the observations are 2 and 5 for $1 - \alpha = 1/3$. It is given that the (upper) 0.3333-quantile of a *t*-distributed stochastic variable with one degree of freedom is $t_{1, \frac{1}{3}} = 1/\sqrt{3} = 0.57735$.
- Comment on whether the interval you have calculated seems reasonable.

Note: You must upload a pdf file here that contains image(s) of your **handwritten** solution to the problem. When correcting this assignment, emphasis will be placed on the answer being **logical** and containing **all natural intermediate calculations.**

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Maximum marks: 15

11B

Introduction: Assume that we observe a random sample consisting of $n = 2$ observations X_1, X_2 from a normal distribution with unknown mean and variance.

Exercise:

- Derive a $(1-\alpha)$-prediction interval for a new observation X_3.
- Compute the interval given that the observations are 12 and 15 and for $1 - \alpha = 1/3$. It is given that the (upper) 0.3333-quantile for a *t*-distributed stochastic variable with one degree of freedom is $t_{1,\frac{1}{3}} = 1/\sqrt{3} = 0.57735$.
- Comment on whether the interval you have calculated seems reasonable.

Note: You must upload a pdf file here that contains image(s) of your **handwritten** solution to the problem. When correcting this assignment, emphasis will be placed on the answer being **logical** and containing **all natural intermediate calculations.**

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Maximum marks: 15

37 **11C**

Introduction: Assume that we observe a random sample consisting of $n = 2$ observations X_1, X_2 from a normal distribution with unknown mean and variance.

Exercise:

- Derive a $(1 - \alpha)$-prediction interval for a new observation X_3
- Compute the interval given that the observations are 1 and 6 and for $1 - \alpha = 1/3$. It is given that the (upper)) 0.3333-quantile for a *t*-distributed stochastic variable with one degree of freedom is $t_{1,\frac{1}{3}} = 1/\sqrt{3} = 0.57735$.
- Comment on whether the interval you have calculated seems reasonable.

Note: You must upload a pdf file here that contains image(s) of your **handwritten** solution to the problem. When correcting this assignment, emphasis will be placed on the answer being **logical** and containing **all natural intermediate calculations.**

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Maximum marks: 15