Oppgave 1

$\bar{X}$: målt promille

$\bar{X} \sim N(\mu, \sigma = 0{,}044)$

↑ sann, objekt promille

a) Antar $\mu = 0{,}2$

$P(0{,}13 \leq \bar{X} \leq 0{,}31)$

$= P(\bar{X} \leq 0.31) - P(\bar{X} \leq 0{,}13)$

$= P\left(Z \leq \frac{0{,}31 - 0{,}2}{0{,}044}\right)$

$- P\left(Z \leq \frac{0{,}13 - 0.2}{0.044}\right)$

$= P(Z \leq 2.5) - P(Z \leq -1.59)$

$= 0{,}9938 - 0.0559$

$= \underline{\underline{0.9379}}$

b) $x_1 = 0{,}22$, $\bar{X}_1 \sim N(\mu, \sigma = 0.044)$

95 % KI ($\alpha = 0.05$)

1. pivotal: $Z = \frac{\bar{X}_1 - \mu}{\sigma}$ ← objekt

↑ antas kjent

2. kvantiler:

$P(z_{1-\frac{\alpha}{2}} \leq Z \leq z_{\alpha/2}) = 1 - \alpha$

3. Løse ulikhetene:

$z_{1-\frac{\alpha}{2}} \leq \frac{\bar{X}_1 - \mu}{\sigma}$

$\sigma z_{1-\frac{\alpha}{2}} \leq \bar{X}_1 - \mu$

$\mu \leq \bar{X}_1 - \sigma z_{1-\frac{\alpha}{2}}$

$\frac{\bar{X}_1 - \mu}{\sigma} \leq z_{\frac{\alpha}{2}}$

$\bar{X}_1 - \sigma z_{\frac{\alpha}{2}} \leq \mu$

4. Sette sammen:

$P(\bar{X}_1 - \sigma z_{\frac{\alpha}{2}} \leq \mu \leq \bar{X}_1 - \sigma z_{1-\frac{\alpha}{2}})$

$= 1 - \alpha$

Husk: $z_{1-\frac{\alpha}{2}} = -z_{\frac{\alpha}{2}}$ i norm.f.

5. Intervall:

$[x_1 - \sigma z_{\frac{\alpha}{2}}, x_1 + \sigma z_{\frac{\alpha}{2}}]$

Tall:

$x_1 \pm z_{\frac{\alpha}{2}} \cdot \sigma$

$= 0{,}22 \pm 1{,}96 \cdot 0.044$

$= \underline{\underline{[0.134, 0{,}306]}}$

c) $n=5$, $\bar{x}=0.22$

$X_i \sim N(\mu, \sigma=0.044)$

$\Rightarrow \bar{X} \sim N(\mu, \sigma/\sqrt{n})$

95% KI:

1. $Z_1 = \dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}}$

2. + 3.: $P(z_{1-\frac{\alpha}{2}} \leq Z_1 \leq z_{\frac{\alpha}{2}})$

$= P\left(z_{1-\frac{\alpha}{2}} \leq \dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \leq z_{\frac{\alpha}{2}}\right) = 1-\alpha$

$z_{1-\frac{\alpha}{2}} \leq \dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ | $\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \leq z_{\frac{\alpha}{2}}$

$\mu \leq \bar{X} - z_{1-\frac{\alpha}{2}} \dfrac{\sigma}{\sqrt{n}}$ | $\bar{X} - z_{\frac{\alpha}{2}} \dfrac{\sigma}{\sqrt{n}} \leq \mu$

$= -z_{\alpha/2}$

$= P\left(\bar{X} - z_{\frac{\alpha}{2}} \dfrac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X} + z_{\frac{\alpha}{2}} \dfrac{\sigma}{\sqrt{n}}\right)$

4. + 5.: $\bar{X} \pm z_{\frac{\alpha}{2}} \dfrac{\sigma}{\sqrt{n}}$

$0.22 \pm 1.96 \pm \dfrac{0.044}{\sqrt{5}}$

$= [0.181, 0.259]$ //

Oppgave 2

X: kvm. pris (i kkr)

$X \sim N(\mu, \sigma)$

c) 95 % KI for μ, $n = 15$

1. pivotal

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_{n-1}$$

2.+3.+4.: kvantiler, løse for μ, sette sammen

$$P(t_{1-\frac{\alpha}{2}, n-1} \leq T \leq t_{\frac{\alpha}{2}, n-1}) = 1 - \alpha$$

$$t_{1-\frac{\alpha}{2}, n-1} \leq \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

$$t_{1-\frac{\alpha}{2}, n-1} \cdot \frac{S}{\sqrt{n}} \leq \bar{X} - \mu$$

$$\mu \leq \bar{X} - t_{1-\frac{\alpha}{2}, n-1} \cdot \frac{S}{\sqrt{n}}$$

$$\frac{\bar{X} - \mu}{S/\sqrt{n}} \leq t_{\frac{\alpha}{2}, n-1}$$

$$\bar{X} - \mu \leq t_{\frac{\alpha}{2}, n-1} \frac{S}{\sqrt{n}}$$

$$\bar{X} - t_{\frac{\alpha}{2}, n-1} \frac{S}{\sqrt{n}} \leq \mu$$

$$t_{1-\frac{\alpha}{2}, n-1} = -t_{\frac{\alpha}{2}, n-1}$$

$$\Rightarrow P\left(\bar{X} - t_{\frac{\alpha}{2}, n-1} \frac{S}{\sqrt{n}} \leq \mu \leq \bar{X} + t_{\frac{\alpha}{2}, n-1} \frac{S}{\sqrt{n}}\right) = 1 - \alpha$$

$$\underline{\underline{\left[\bar{X} - t_{\frac{\alpha}{2}, n-1} \frac{S}{\sqrt{n}}, \bar{X} + t_{\frac{\alpha}{2}, n-1} \frac{S}{\sqrt{n}}\right]}}$$

$n = 15$, $\bar{x} = 32$, $\sum(x_i - \bar{x})^2 = 74.1$

$$S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$$

$$t_{\frac{\alpha}{2}, n-1} = t_{0.025, 14} = 2.145$$

$$\bar{x} \pm t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}}$$

$$= \bar{x} \pm t_{\frac{\alpha}{2}, n-1} \cdot \frac{1}{\sqrt{n}} \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$$

$$= 32 \pm 2.145 \cdot \frac{1}{\sqrt{15}} \cdot \sqrt{\frac{74.1}{14}}$$

$$= \underline{\underline{[30.7, 33.3]}}$$

Oppgave 3

$T \sim Exp(\beta)$

$$= \begin{cases} \frac{1}{\beta} e^{-t/\beta}, & t \geq 0 \\ 0, & \text{ellers} \end{cases}$$

$E(T) = \beta$

c) Først: fra 2b):

$$\frac{2}{\beta} T \sim \chi^2_2$$

$T_1, \ldots, T_n$ uavh.

$\Rightarrow \frac{2}{\beta} T_i$ uavh.

$$\frac{2}{\beta} \sum_{i=1}^{n} T_i = \frac{2}{\beta} T_1 + \frac{2}{\beta} T_2 + \ldots$$

$= \text{sum av } n\ \chi^2_2\text{-variabler}$

$$= \chi^2_{\sum_{i=1}^{n} 2} = \chi^2_{2n}$$

$(1-\alpha)100\,\%$ KI for β

1. $\frac{2}{\beta} \sum_{i=1}^{n} T_i \sim \chi^2_{2n}$

2.+3.+4.:

$$P\left(X_{1-\frac{\alpha}{2},2n} \leq \frac{2}{\beta} \sum T_i \leq X_{\frac{\alpha}{2},2n}\right) = 1-\alpha$$

$$X_{1-\frac{\alpha}{2}} \leq \frac{2}{\beta} \sum T_i$$

$$\beta \leq \frac{2}{X_{1-\frac{\alpha}{2}}} \sum T_i$$

$$\frac{2}{\beta} \sum T_i \leq X_{\frac{\alpha}{2}}$$

$$\frac{2}{X_{\alpha/2}} \sum T_i \leq \beta$$

$$P\left(\frac{2}{X_{\frac{\alpha}{2}}} \sum T_i \leq \beta \leq \frac{2}{X_{1-\frac{\alpha}{2}}} \sum T_i\right)$$

$$= 1-\alpha$$

5.

$$\left[\frac{2}{X_{\frac{\alpha}{2},2n}} \sum_{i=1}^{n} t_i, \frac{2}{X_{1-\frac{\alpha}{2},2n}} \sum_{i=1}^{n} t_i\right]$$

$95\,\%$ KI $\Rightarrow \alpha = 0.05$

$n = 20$, $\sum_{i=1}^{20} t_i = 30$

$$X_{\frac{\alpha}{2},2n} = X_{0.025,40} = 59.342$$

$$X_{1-\frac{\alpha}{2},2n} = X_{0.975,40} = 24.433$$

$$\left[\frac{2}{59.342} \cdot 30, \frac{2}{24.433} \cdot 30\right]$$

$$= \underline{\underline{[1.011, 2.456]}}$$