OPPGAVE 3

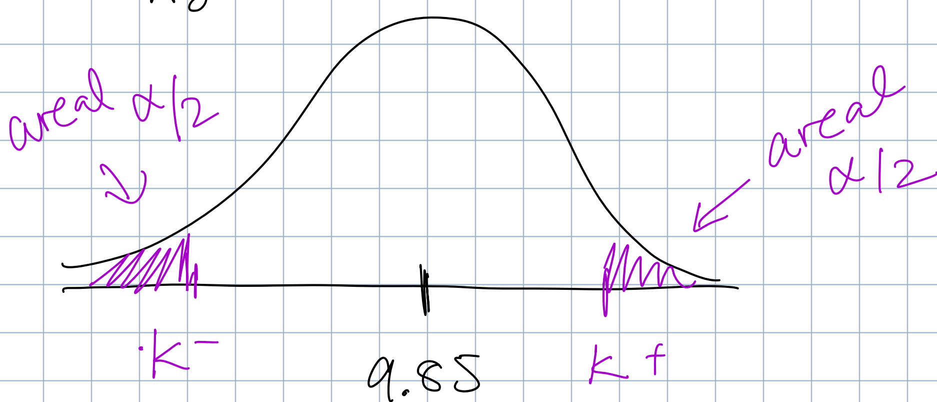
a) $H_0: \mu = 9.85 \qquad H_1: \mu \neq 9.85$

b) $H_0: \sigma \leq 0.05 \qquad H_1: \sigma > 0.05$

$\rightarrow \quad \sigma^2 \leq 0.05^2 \qquad H_1: \sigma^2 > 0.05^2$

c) For μ: $\quad \bar{X} \sim N\left(\mu, \frac{\sigma^2}{10}\right)$

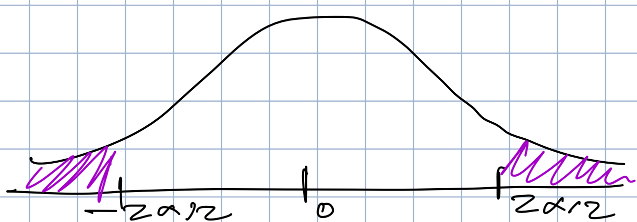
H_0:



P(forkaste H_0 når H_0 er sann) $= \alpha/2 + \alpha/2 = \alpha$

→ standardisert testobservator

$$Z = \frac{\bar{X} - 9.85}{\sigma/\sqrt{10}} \overset{H_0}{\sim} N(0,1)$$



Men vi kjenner ikke σ (!) $\longrightarrow$ sett inn S

$$T = \frac{\bar{x} - 9.85}{S/\sqrt{10}} \sim t_9$$

$\uparrow$

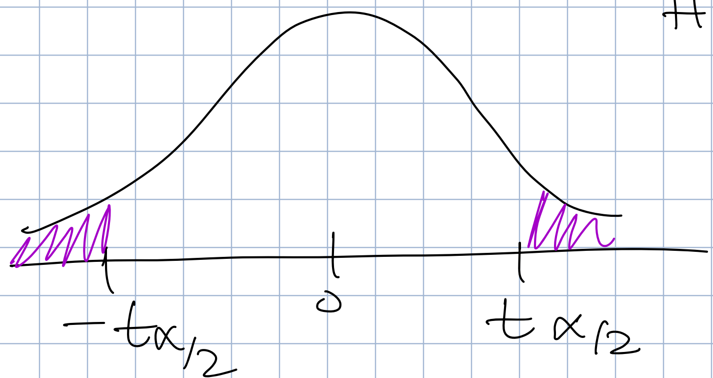
H_0

Finn kritisk verdi

$\swarrow \alpha/2$

her $\nu = 9$, "α" $= 0.025$

$t_{\alpha/2} = 2.262$



Alarm bør gå for μ dersom vi observerer

$t > 2.262$ eller $t < -2.262$

(5% sjanse under H_0)

For σ^2: Ta utgangspunkt i $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X})^2$

Husk $\frac{(n-1)}{\sigma^2} S^2 \sim \chi^2_{n-1}$

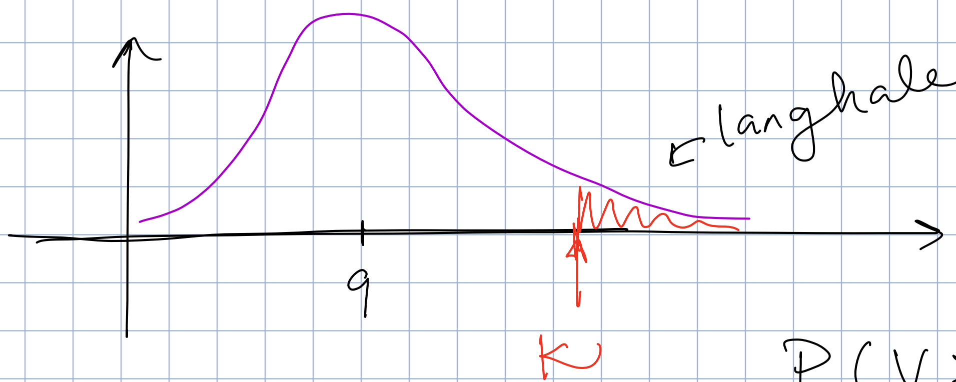
H_0 $\quad V = \frac{n-1}{0.05^2} S^2 \sim \chi^2_9$

$S^2 \approx 0.05^2$

$\rightarrow V \approx n-1 = 9$

$S^2 > 0.05^2$

$\rightarrow$ store tale



$P(V > k) = 0.05$

$k = 16.919$

d) anta $\sigma = 0.05$ (kjent, ingen endring)

↳ kan bruke

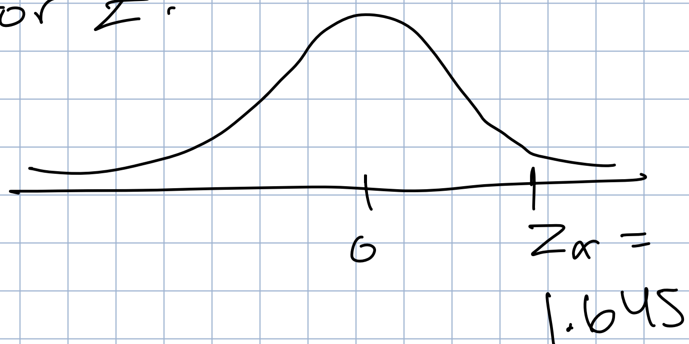
$$Z = \frac{\bar{X} - 9.85}{\sigma/\sqrt{10}} \underset{H_0}{\sim} N(0,1)$$

i stedet for

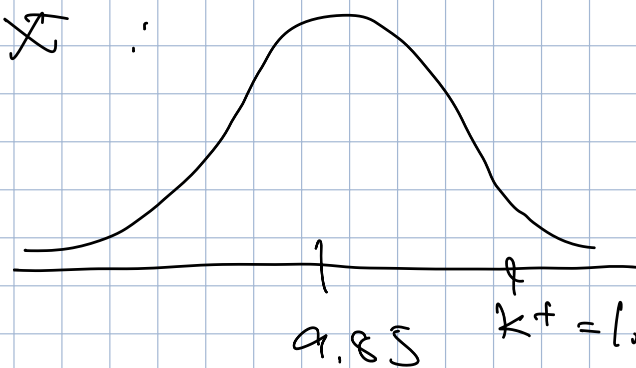
$$T = \frac{\bar{X} - 9.85}{S/\sqrt{10}} \underset{H_0}{\sim} t_9$$

Forkastningsregel:

For Z:



For $\bar{X}$:



$k^* = 1.645 \cdot \frac{0.05}{\sqrt{10}} + 9.85$

$= 9.876$

e) Teststyrke = P(forkaste H_0 når H_1 sann)

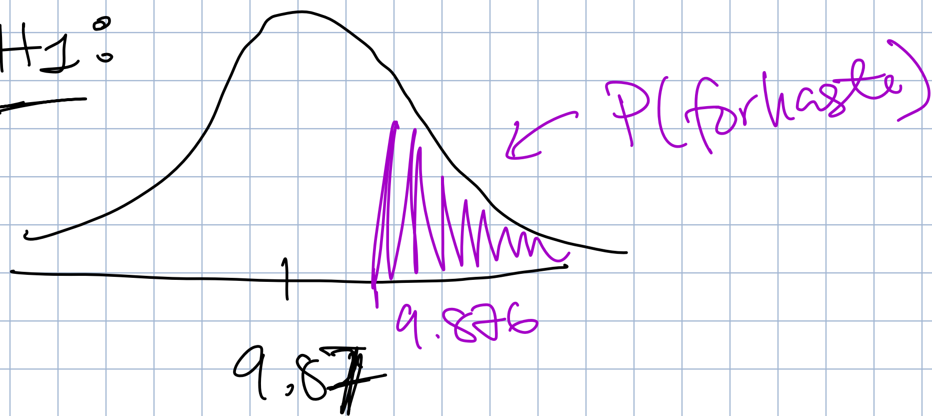
$(= 1 - P(\text{type 2 feil}))$

$P(Z \geq 1.645)$ når $\bar{X} \sim N(9.87, \frac{0.05}{\sqrt{10}})$

$=$

$P(\bar{X} \geq 9.876)$ når $\bar{X} \sim N(9.87, \frac{0.05}{\sqrt{10}})$

H1:



$$P(\bar{X} \geq 9.876) = 1 - P(\bar{X} \leq 9.876)$$

$$= 1 - P\left(Z \leq \frac{9.876 - 9.87}{0.05/\sqrt{10}}\right)$$

$$= 1 - P(Z \leq 0.38) = 1 - 0.6480$$

$$= 0.352$$

f) "Ukjent n"

NB $\quad k = 9{,}85 + 1{,}645 \frac{\sigma}{\sqrt{n}}$

$P(\text{forkaste } H_0 \text{ når } \mu = 9.87) = P(\bar{X} \geq k)$

for $\bar{X} \sim N(9.87, \frac{\sigma}{\sqrt{n}})$

$P(\bar{X} \geq k) =$

$P\left(Z \geq \frac{k - 9.87}{\sigma/\sqrt{n}}\right) \geq 0.6$

$$\Downarrow$$

$$P\left(Z \leq \frac{k-9.87}{\sigma/\sqrt{n}}\right) \leq 0.4$$ (så nært som mulig)

tabell : $$\frac{k-9.87}{\sigma/\sqrt{n}} \approx -0.26$$

$$\frac{9.85+1.645\frac{\sigma}{\sqrt{n}}-9.87}{\sigma/\sqrt{n}} \approx -0.26$$

$$1.645 - \frac{0.02}{\sigma/\sqrt{n}} \approx -0.26$$

$$1.905 \approx \frac{0.02}{\sigma} \cdot \sqrt{n}$$

$$\sqrt{n} \approx 1.905 \cdot \frac{0.05}{0.02} = 4.76$$

$$\Rightarrow n \approx 22.66$$

$$n = 23$$