I) Feilforplantning (forv. og varians)

← Sann lengde

L : måling av lengde $\quad E(L) = l \quad Var(L) = \sigma_l^2$

B : måling av bredde $\quad E(B) = b \quad Var(B) = \sigma_b^2$

← sann bredde

$\boxed{a = l \cdot b}$ Sant areal

$A = L \cdot B \qquad E(A) = ? \qquad Var(A) = ?$

gjetning på l $\quad$ gjetning på b

"Tenkte observasjoner" $\quad x_l \pm \sigma_l \qquad x_b \pm \sigma_b$

$\boxed{x_a \pm \sigma_a}$ ← hva skal stå her?

standardavvik i A, approksimert.

gjetning på a, $E(A)$

← målefunksjon

$$x_a = g(x_l, x_b) = x_l x_b$$

$$\frac{\partial x_a}{\partial x_l} = x_b \qquad \frac{\partial x_a}{\partial x_b} = x_l$$

I. ordens taylorapproksimasjon rundt sanne verdier l, b:

$$x_a \approx g(l,b) + \frac{\partial x_a}{\partial x_l}(l,b)\cdot(\underline{x_l} - l) + \frac{\partial x_a}{\partial x_b}(l,b)(\underline{x_b} - b)$$

nå er x_a lineær i x_l og x_b

$$= lb + b\cdot(x_l - l) + l\cdot(x_b - b)$$

STOK. VAR:

$$A \approx lb + b(L - l) + l\cdot(B - b)$$

$$E(A) \approx lb + b(E(L) - l) + l(E(B) - b)$$

$\Rightarrow E(A) \approx l \cdot b$ $\quad\quad [E(A) \approx E(L) \cdot E(B)]$

NB: Siden vi antar L og B uavhengige, så vet vi at $E(LB) = E(L)E(B)$, approksimasjonen er derfor eksakt

---

$$Var(A) \approx 0 + b^2 Var(L) + l^2 Var(B)$$

$$= b^2 \sigma_L^2 + l^2 \sigma_B^2$$

kalles noen ganger følsomhetsfaktorer

---

siden $l >> b$, vil σ_B (som ganges med l^2) ha mest å si.

c) $X_a = X_1 \cdot X_b \pm \sqrt{X_b^2 \sigma_1^2 + X_1^2 \sigma_b^2}$

$$= 10 \cdot 0.8 \pm \sqrt{0.8^2 \cdot 0{,}05^2 + 10^2 \cdot 0{,}01^2}$$

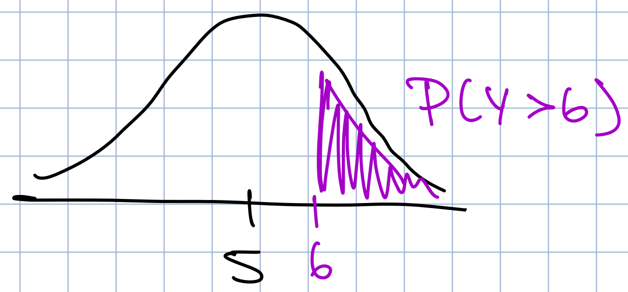
$$= 8 \pm \sqrt{0.0016 + 0.01}$$

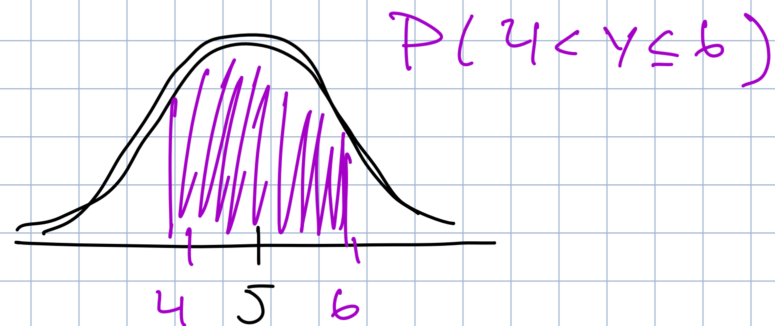
$$= 8 \pm 0{,}11 \ m^2$$

2. Regresjon og hyp.testing

Y : biomasse av en plante

$Y \sim N(5, 4)$





$$P(Y>6) = 1 - P(Y \leq 6) = 1 - P\left(Z \leq \frac{6-5}{2}\right)$$

$$= 1 - \Phi(0.5) = 1 - 0.6915 = 0.3085$$

$$P(4 < Y \leq 6) = P(Y \leq 6) - P(Y \leq 4) = 0.6915 - 0.3085$$

$$= 0.3830$$

$$P(Y>6 \mid Y>4) = \frac{P(Y>6 \cap Y>4)}{P(Y>4)} = \frac{P(Y>6)}{P(Y>4)}$$

$$= \frac{0.3085}{0.6915} \approx 0.45$$

Lin reg $\quad Y = \beta x + \varepsilon(x) \quad \varepsilon(x) \sim N(0, \underline{\tau^2 x^2})$

variasjon i biomasse øker med kultiveringsperiode

$Y \sim N(\beta x, \tau^2 x^2)$

$(y_i, x_i) \quad i = 1, \dots, 5$

$$L(\beta, \tau^2) = \prod_{i=1}^{5} f(y_i) = \prod_{i=1}^{5} \frac{1}{\sqrt{2\pi \tau^2 x_i^2}} \exp\left(-\frac{(y_i - \beta x_i)^2}{2\tau^2 x_i^2}\right)$$

$$l(\beta,\tau^2) = \sum_{i=1}^{5} \ln\left(\frac{1}{\sqrt{2\pi\tau^2 x_i^2}}\right) - \frac{1}{2\tau^2 x_i^2}(y_i-\beta x_i)^2$$

$$\frac{\partial l}{\partial \beta} = \sum_{i=1}^{5} 0 \cancel{-} \frac{1}{\cancel{2}\tau^2 x_i^{\cancel{2}}} \cdot \cancel{2}(y_i-\beta x_i)(\cancel{-}\cancel{x_i})$$

$$= \sum_{i=1}^{5} \frac{1}{\tau^2 x_i}(y_i-\beta x_i) = \sum_{i=1}^{5} \frac{1}{\tau^2}\left(\frac{y_i}{x_i} - \beta\right)$$

Løs $\frac{\partial l}{\partial \beta} = 0$ $\qquad \frac{1}{\cancel{\tau^2}} \sum_{i=1}^{5}\left(\frac{y_i}{x_i} - \beta\right) = 0$

$$\sum \frac{y_i}{x_i} - 5\beta = 0 \qquad \beta = \frac{1}{5}\sum_{i=1}^{5}\frac{y_i}{x_i}$$

$$\hat{\beta} = \frac{1}{5}\sum_{i=1}^{5}\frac{Y_i}{x_i}$$ ← stokastisk

Finn $\hat{\tau}^2$ selv, se LF.

Estimat $\hat{\beta} = \frac{1}{5}\left(\frac{1}{3} + \frac{5}{6} + \frac{3}{7} + \frac{3}{10} + \frac{10}{14}\right)$

$= 0.52$

$H_0 : \beta = 0.50 \qquad H_1 : \beta > 0.50$

Naturlig testobservator $\hat{\beta}$:

$\hat{\beta} = \frac{1}{5} \sum_{i=1}^{5} \frac{Y_i}{x_i}$ hvilken fordeling har denne under H_0?

$Y_i \sim N(\beta x_i, \tau^2 x_i^2) \Rightarrow \frac{Y_i}{x_i} \sim N(\beta, \tau^2)$

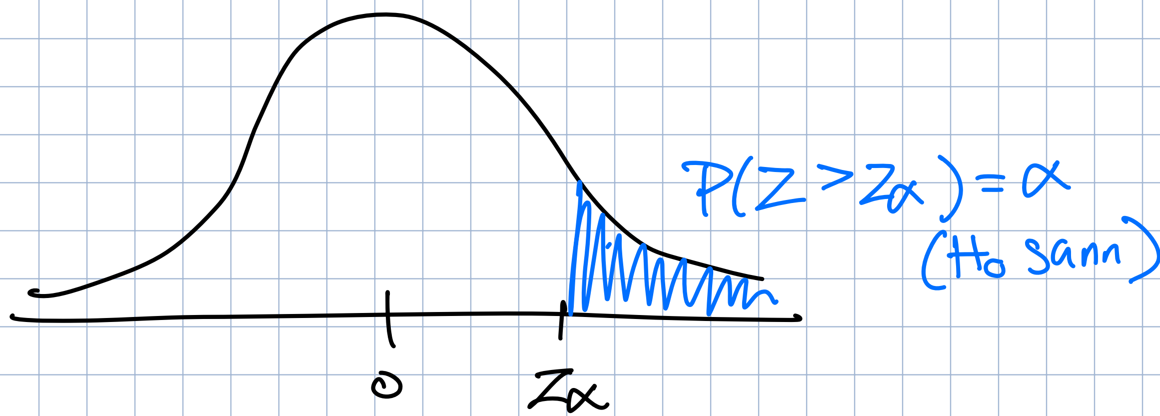
utbrodere ?

$\hat{\beta}$ sum av uavh. normalfordelt → normalfordelt

$$E(\hat{\beta}) = \frac{1}{5} \sum_{i=1}^{5} E\left(\frac{y_i}{x_i}\right) = \beta$$

$$Var(\hat{\beta}) = \frac{1}{5^2}\, 5 \cdot \tau^2 = \tau^2/5$$

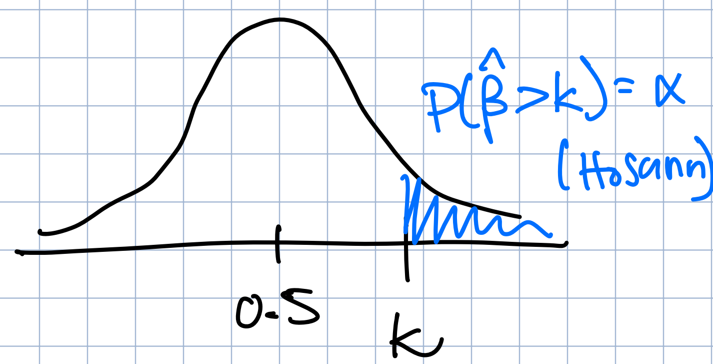
$$Z = \frac{\hat{\beta} - 0.5}{\tau/\sqrt{5}} \overset{H_0}{\sim} N(0,1)$$



$$Z_{0.05} = 1.645$$

Eller

$$\hat{\beta} \overset{H_0}{\sim} N(0.5, \tau/\sqrt{5})$$



$$k = 0.5 + Z_{0.05} \frac{\tau}{\sqrt{5}}$$

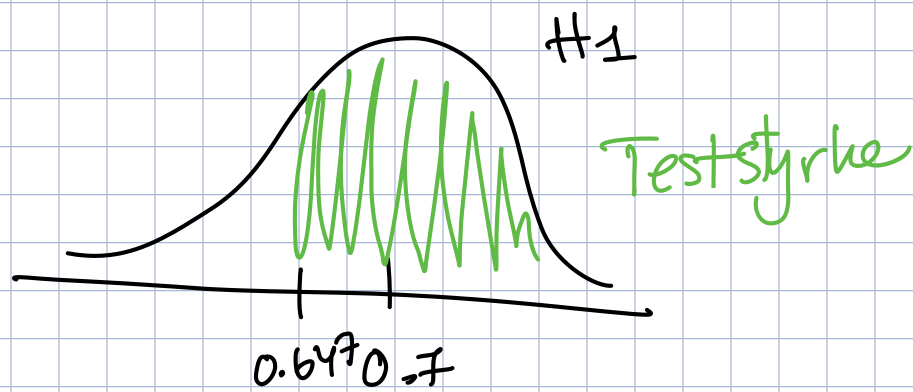
$$= 0.5 + 1.645 \cdot \frac{0.4}{\sqrt{5}}$$

$$= 0.647$$

Est $\hat{\beta} = 0.5$, ikke forkast.

$\hat{\beta} \sim N(0.7, \tau^2/5)$

Teststyrke: $P(\hat{\beta} > 0.647 \mid H_1 \text{ sann})$



$$= 1 - P(\hat{\beta} \leq 0.647)$$

$$= 1 - P\left(\frac{\hat{\beta} - 0.7}{\tau/\sqrt{5}} \leq \frac{0.647 - 0.7}{\tau/\sqrt{5}}\right)$$

$$= 1 - P(Z \leq -0.59) = 1 - 0.2776 \approx \underline{\underline{0.72}}$$