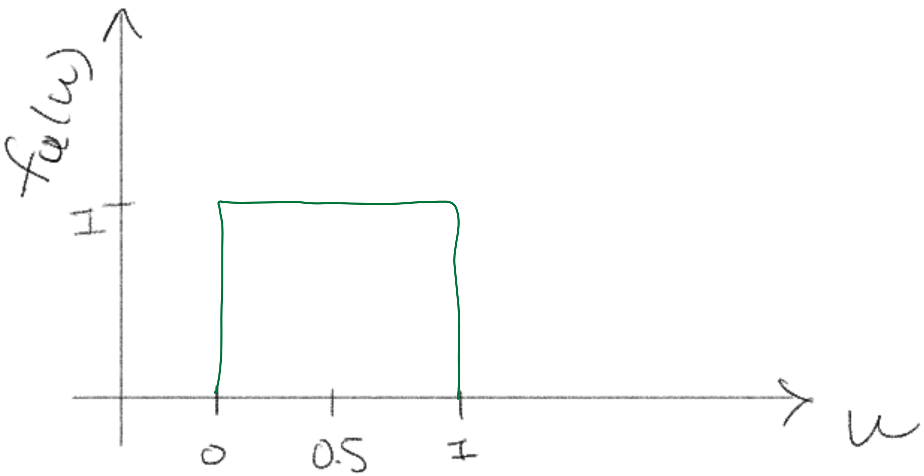
OPPG. 1

$U \sim \text{unif}(0,1)$



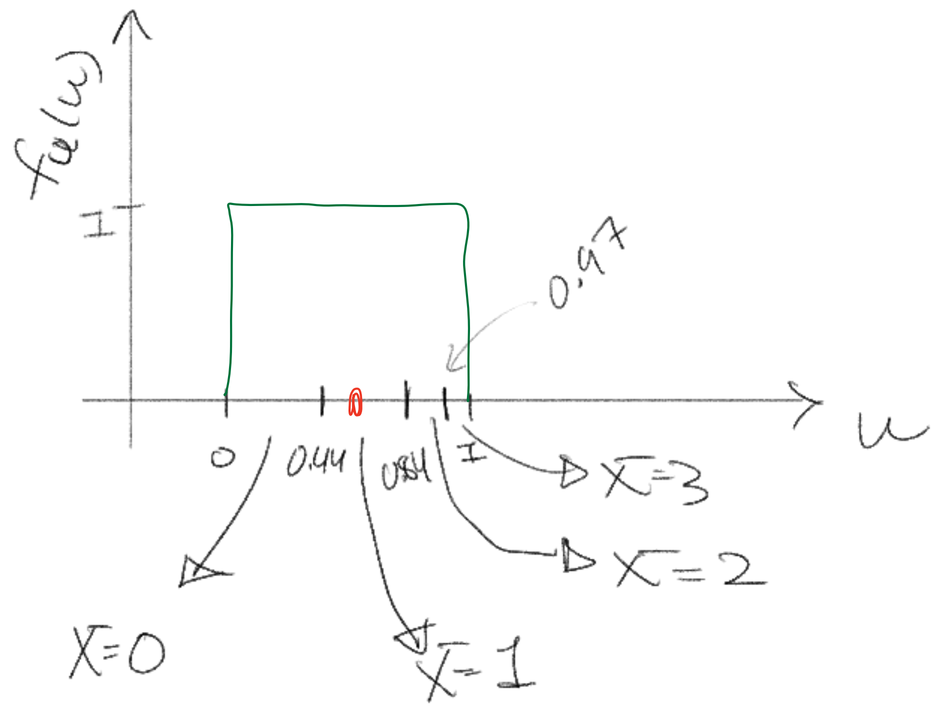
$$f_U(u) = \begin{cases} 1 & \text{for } 0 \le u \le 1 \\ 0 & \text{ellers} \end{cases}$$

$$\int_0^1 1 \, du = [u]_0^1 = 1$$

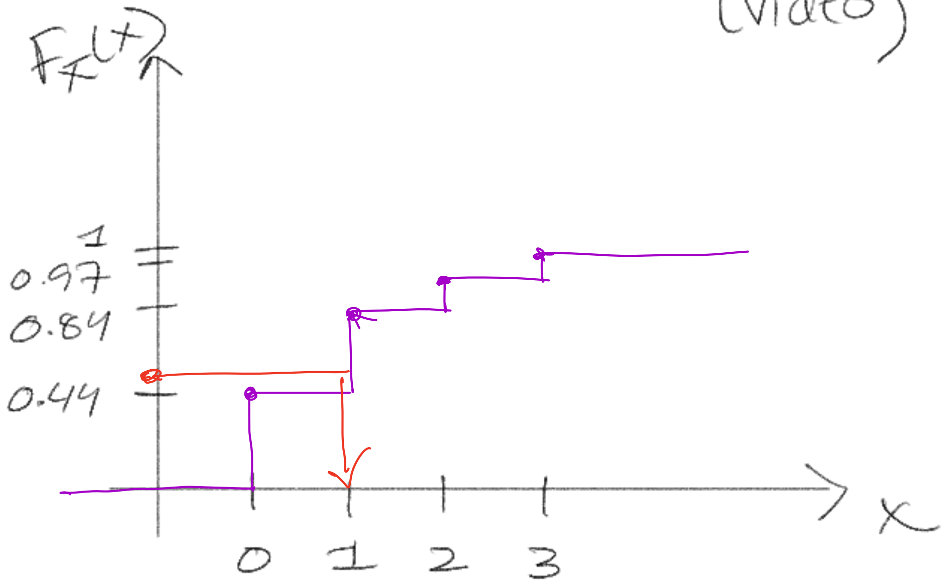
Eks: $P(0 \le U \le 0.5) = 0.5$

$P(0.5 \le U \le 0.8) = 0.3$

og $P(0 \le U \le 0.44) = 0.44$



(video)



$P(X=0) = P(0 \leq U \leq 0.44) = 0.44$

$P(X=1) = P(0.44 \leq U \leq 0.84) = 0.40$

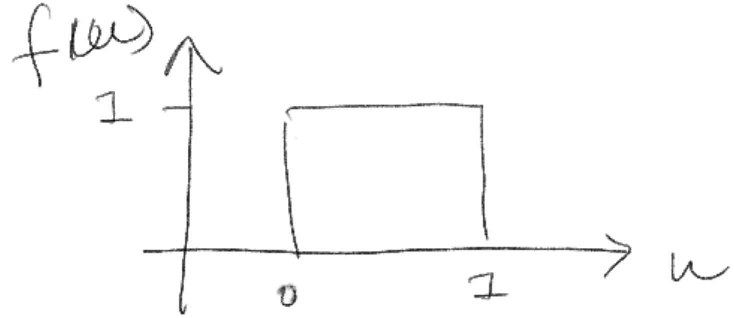
$P(X=2) = P(0.84 \leq U \leq 0.97) = 0.13$

$P(X=3) = P(0.97 \leq U \leq 1) = 0.03$

$u = 0.52 \quad x = 1$

[PYTHON]

OPPG. 2a



$U \sim \text{Unif}(0,1)$

$Y = \varphi \cdot \sqrt{U}$

- hva er $f_Y(y)$?

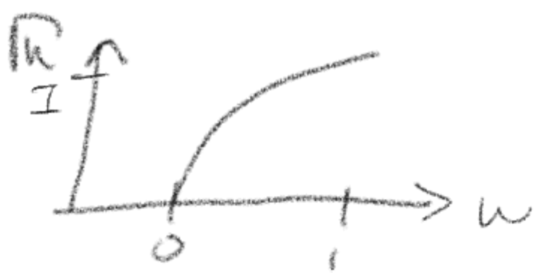
NB: Jobb med kumulative sanns. først!

(1) Hva er $F_U(u) = P(U \leq u)$?

Svar: $F_U(u) = \int_0^u 1 \, du = [u]_0^u = u$

(2) $\sqrt{0} = 0 \quad \sqrt{1} = 1$

$y \in [0, \varphi]$



(3) $P(Y \leq y) = P(\varphi\sqrt{U} \leq y)$

$$= P\left(\sqrt{U} \leq \frac{y}{\varphi}\right)$$

$$= P\left(U \leq \left(\frac{y}{\varphi}\right)^2\right)$$

Husk $P(U \leq u) = u$

$$= \frac{y^2}{\varphi^2} \quad \text{for} \quad 0 \leq y \leq \varphi$$

(4) $f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{d}{dy} \frac{y^2}{\varphi^2} = \frac{2y}{\varphi^2}$

Eks: $\varphi = 5$

$$f_Y(y) = \frac{2y}{25} \quad \text{for} \quad 0 \leq y \leq 5$$

$$= 0 \quad , \quad \text{ellers}$$

[PYTHON]

OPPG. 2b

$$F_X(x) = P(X \le x) = 1 - e^{-x^2/\theta} \quad \text{for} \quad x \ge 0$$

$$X = g(U) \qquad U \sim \text{Unif}(0, 1)$$

hva er g ?

$$X = F_X^{-1}(U) \quad \leftarrow g \text{ er } F_X^{-1}(U)$$

$$u = F_X(x) = 1 - e^{-x^2/\theta}$$

$$1 - u = e^{-x^2/\theta}$$

$$\ln(1-u) = -x^2/\theta$$

$$\theta \ln(1-u) = -x^2$$

$$x^2 = -\theta \ln(1-u)$$

$$x = \sqrt{-\theta \ln(1-u)}$$

$$X = \sqrt{-\theta \ln(1-U)}$$

Python

```
import numpy as np

def simX(n, theta)
    u=np.random.uniform(size=n)
    x= np.sqrt(-theta*np.log(1-u))

    return x
```

Kan overbevise oss om at svaret ble rett

$$X = \sqrt{-\theta \ln(1-U)} \quad \text{der} \quad U \sim \text{Unif}(0,1)$$

Hva er $F_X(x)$?

$$P(X \leq x) = P\left(\sqrt{-\theta \ln(1-U)} \leq x\right)$$

$$= P(-\theta \ln(1-U) \leq x^2)$$

$$= P\left(\ln(1-U) \geq \frac{-x^2}{\theta}\right)$$

$$= P\left(1-U \geq \exp\left(-\frac{x^2}{\theta}\right)\right)$$

$$= P\left(-U \geq -1 + \exp\left(-\frac{x^2}{\theta}\right)\right)$$

$$= P\left(U \leq 1 - \exp\left(\frac{-x^2}{\theta}\right)\right) = F_U\left(1-\exp\left(\frac{-x^2}{\theta}\right)\right) = 1-\exp\left(\frac{-x^2}{\theta}\right)$$