Oppgave 1

T: levetid i timer

$T \sim Exp(\beta) = \frac{1}{\beta} e^{-t/\beta}$, $t \geq 0$ (0 ellers)

OBS! $E(T) = \beta$

a) $\beta = 30$

i)

$$P(T \leq t) = \int_{-\infty}^{t} f(x)\,dx$$

$$= \int_{0}^{t} \frac{1}{\beta} e^{-x/\beta}\,dx$$

$$= \frac{1}{\beta}\left[-\beta e^{-\frac{x}{\beta}}\right]_0^t$$

$$= \frac{1}{\cancel{\beta}}\left(-\cancel{\beta} e^{-t/\beta} + \cancel{\beta}\right)$$

$$= \underline{1 - e^{-t/\beta}}$$

$\beta = 30$

$\Rightarrow P(T \leq t) = \underline{\underline{1 - e^{-\frac{t}{30}}}}$, $t \geq 0$ (0 ellers)

ii)

$$P(T < 20) = 1 - e^{-\frac{20}{30}}$$

$$\approx 1 - 0.5134$$

$$= \underline{\underline{0.4866}}$$

$$P(T < 20 \cup T > 40)$$

$$= P(T<20) + P(T>40)$$

$$= 0.4866 + (1 - P(T<40))$$

$$= 0.4866 + \left(1 - \left(1 - e^{-\frac{40}{30}}\right)\right)$$

$$\approx 0.4866 + 0.2636$$

$$= \underline{\underline{0.7502}}$$

iii)

$$P(T \leq k) = 0.5$$

$$1 - e^{-k/30} = 0.5$$

$$e^{-k/30} = 0.5$$

$$-\frac{k}{30} = \ln(0.5)$$

$$k = -30\ln(0.5)$$

$$\approx \underline{\underline{20.79}}$$

b)

$$P(T \geq t+s \mid T \geq t) = P(T \geq s),\ s \geq 0$$

$P(T \geq t+s \mid T \geq t)$

$= \dfrac{P(T \geq t+s \cap T \geq t)}{P(T \geq t)}$

$= \dfrac{P(T \geq t+s)}{P(T \geq t)} = *$

$P(T \geq t) = 1 - F(t)$

$= 1 - (1 - e^{-t/\beta}) = e^{-t/\beta}$

$* = \dfrac{e^{-(t+s)/\beta}}{e^{-t/\beta}} = \dfrac{\cancel{e^{-t/\beta}} e^{-s/\beta}}{\cancel{e^{-t/\beta}}}$

$= e^{-s/\beta} = P(T \geq s)$ $\square$

(Ingen oppgave 2 !! ☺)

<u>Oppgave 3a</u>

Z_k: tidspkt. for hendelse nr. k i Poissonprosess m/ intensitet λ

$X_1 = Z_1, \; X_k = Z_k - Z_{k-1}$

$k = 2, 3, \ldots$

X_1: tid til 1. hendelse

X_k: tid mellom to hendelser

Vet da:

$X_1, X_2, \ldots \sim Exp(\lambda)$

OBS! $E(X_i) = 1/\lambda$

$P(X_1 \geq 2) = 1 - P(X_1 \leq 2)$

$= 1 - F_{X_1}(2) = 1 - (1 - e^{-2\lambda})$

$= \underline{\underline{e^{-2\lambda}}}$

$P(X_1 + X_2 \geq 4)$

<u>Versjon 1</u>

X_1: tid til 1. hendelse

X_2: tid mellom 1. og 2. hendelse

$X_1 + X_2$: tid til 2. hendelse

N(t): teller antall hendelser i intervallet $[0,t]$

$$\Rightarrow N(t) \sim Poi(\lambda t)$$

$$= \frac{(\lambda t)^n}{n!} e^{-\lambda t}$$

$$P(X_1 + X_2 \geq 4)$$

$$= P(N(4)=0 \cup N(4)=1)$$

$$= P(N(4)=0) + P(N(4)=1)$$

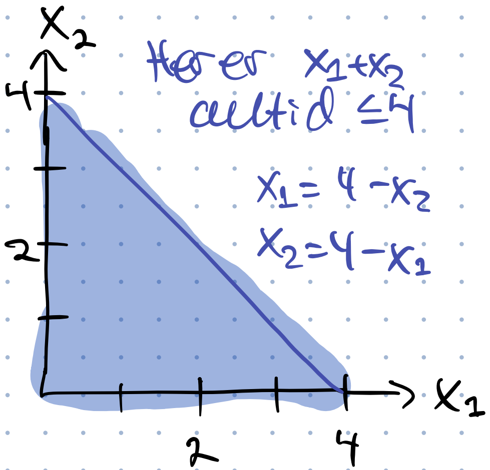
$$= \frac{(4\lambda)^0}{0!} e^{-4\lambda} + \frac{(4\lambda)^1}{1!} e^{-4\lambda}$$

$$= e^{-4\lambda} + 4\lambda e^{-4\lambda} = \underline{\underline{e^{-4\lambda}(1+4\lambda)}}$$

<u>Versjon 2</u>

$$P(X_1 + X_2 \geq 4) = 1 - P(X_1 + X_2 \leq 4)$$

$$1 - \iint f_{X_1,X_2}(x_1, x_2)\,dx_1\,dx_2$$



Grenser: $x_2: 0 \rightarrow 4 - x_1$

$x_1: 0 \rightarrow 4$

$$= 1 - \int_0^4 \int_0^{4-x_1} f(x_1) f(x_2)\,dx_2\,dx_1$$

$$\left.\begin{array}{l} X_1 \sim Exp(\lambda) \\ X_2 \sim Exp(\lambda) \end{array}\right\} = \lambda e^{-\lambda x_i} \quad x_i \geq 0$$

$$= 1 - \int_0^4 \int_0^{4-x_1} \lambda e^{-\lambda x_1} \lambda e^{-\lambda x_2}\,dx_2\,dx_1$$

$$= 1 - \int_0^4 \lambda e^{-\lambda x_1} \int_0^{4-x_1} \lambda e^{-\lambda x_2}\,dx_2\,dx_1$$

$$\int_0^{4-x_1} \lambda e^{-\lambda x_2}\,dx_2 = \left[-e^{-\lambda x_2}\right]_0^{4-x_1}$$

$$= -e^{-\lambda(4-x_1)} + 1 = 1 - e^{-4\lambda} \cdot e^{\lambda x_1}$$

$$= 1 - \int_0^4 \lambda e^{-\lambda x_1}\left(1 - e^{-4\lambda} \cdot e^{\lambda x_1}\right) dx_1$$

$$= \dots = \underline{\underline{e^{-4\lambda}(1+4\lambda)}}$$

Version 3

$$X_1 + X_2 \sim \text{Gamma}\left(\alpha = 2, \beta = \frac{1}{\lambda}\right)$$

$$k = 2, \quad \lambda = \lambda$$

$$P(X_1 + X_2 \geq 4) =$$

$$\sum_{n=0}^{1} \frac{1}{n!} e^{-\lambda 4} (\lambda 4)^n$$

$$= \underline{1} \cdot e^{-4 \cdot \lambda} \cdot \underline{1} + \underline{1} \cdot e^{-4\lambda} \cdot 4\lambda$$

$$= \underline{\underline{e^{-4\lambda}(1 + 4\lambda)}}$$

Uten wikipedia:

$$\text{Gamma}\left(\alpha = 2, \beta = \frac{1}{\lambda}\right) = f(x)$$

$$= \frac{\lambda^2}{\Gamma(2)} x^{2-1} e^{-x \cdot \lambda}$$

$$= \lambda^2 x e^{-\lambda x}, \quad x \geq 0$$

(0 ellers)

$$P(X_1 + X_2 \geq 4)$$

$$= \underline{1} - \int_0^4 f(x)\, dx$$

$$= 1 - \int_0^4 \lambda^2 x e^{-\lambda x}\, dx$$

$$= \ldots$$

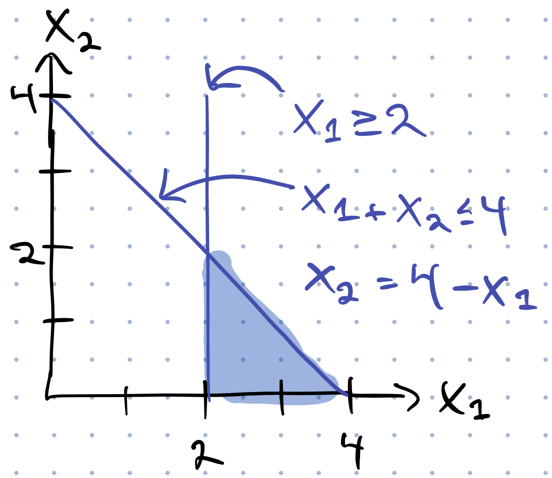
$$= \underline{1} - \cancel{\lambda^2} \left[- \frac{e^{-\lambda x}(\lambda x + 1)}{\cancel{\lambda^2}} \right]_0^4$$

$$= \underline{1} + e^{-4\lambda}(4\lambda + 1) - \underline{1}$$

$$= \underline{\underline{e^{-4\lambda}(4\lambda + 1)}}$$

$P(X_1 + X_2 \geq 4 \mid X_1 \geq 2)$

$= 1 - P(X_1 + X_2 \leq 4 \mid X_1 \geq 2)$



$X_1 : 2 \to 4$

$X_2 : 0 \to 4 - X_1$