OPPG. 1 a

Finn $M_X(t) = E(e^{tX})$ Husk $E(g(X)) = \sum_x g(x) P(X=x)$

$$E(e^{tX}) = \sum_{x=0}^{2} e^{tx} P(X=x)$$

$$= e^{t0} \frac{1}{2} + e^{t \cdot 1} \frac{1}{5} + e^{t \cdot 2} \frac{3}{10}$$

$$= \frac{1}{10} (5 + 2e^t + 3e^{2t})$$

OPPG. 1b

* Finn $E(X)$ med $M_X(t)$

$$M_X'(t) = \frac{1}{10} (0 + 2e^t + 2 \cdot 3e^{2t})$$

$$M_X'(0) = \frac{1}{10} (2e^0 + 6e^0) = 8/10 = \underline{\underline{0.8}}$$

OPPG. 2

STACK 3, oppg 4, togtider

Før:

a) $E(X) = \int_{-\infty}^{\infty} x f_X(x)\, dx$

$= \int_0^{\infty} x \lambda e^{-\lambda x} dx$

delvis integrasjon

$\lim_{x\to\infty} x/e^{\lambda x}$

$= \frac{1}{\lambda}$

$M_X'(t) = ? \quad (-1) \cdot \frac{1}{(1-t/\lambda)^2} \cdot \left(-\frac{1}{\lambda}\right) = \frac{1/\lambda}{(1-t/\lambda)^2}$

$M_X'(0) = ? = \frac{1/\lambda}{(1-0)^2} = \underline{\underline{1/\lambda}}$

b) $E(X^2) = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx = \int_0^{\infty} x^2 \lambda e^{-\lambda x}\,dx$

må bruke delvis integrasjon flere ganger...

Med MGF:

$M_X''(t) = ? \quad = \frac{d}{dt} \frac{1/\lambda}{(1-t/\lambda)^2} = (-2)\frac{1/\lambda}{(1-t/\lambda)^3}(-1/\lambda) = \frac{2(1/\lambda)^2}{(1-t/\lambda)^3}$

$M_X''(0) = ? \quad = 2\frac{(1/\lambda)^2}{(1-0)^3} = \frac{2}{\lambda^2} = E(X^2)$ "andre moment"

$$Var(X) = E(X^2) - E(X)^2 = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 = \frac{2-1}{\lambda^2} = \underline{\underline{\frac{1}{\lambda^2}}}$$

OPPG. 3

Husk at vi utledet i forelesningen om Poisson at summen av to uavhengige Poisson fordelinger også var Poissonfordelt

REP: $X_A \sim \text{Poisson}(\lambda_A t)$ $\quad X_B \sim \text{Poisson}(\lambda_B t)$

$$Y = X_A + X_B \longrightarrow Y \sim \text{Poisson}((\lambda_A + \lambda_B)t)$$

Nå gjør vi dette mer generelt vha MGF.

$X_i \sim \text{Poisson}(\lambda_i)$ $\quad i = 1, \dots, n$ $\quad$ uavhengige

$$Y = \sum_{i=1}^{n} X_i$$

Først: MGF til $\sum_{i=1}^{n} X_i$:

$$M_{\sum X_i}(t) \overset{\text{def}}{=} E(e^{t \sum X_i}) = E\left(\prod_{i=1}^{n} e^{tX_i}\right) \overset{\text{uavh. } X\text{-er}}{=} \prod_{i=1}^{n} E(e^{tX_i})$$

$$\overset{\text{def}}{=} \prod_{i=1}^{n} M_{X_i}(t)$$

Finn $M_Y(t) = M_{\sum X_i}(t) = \prod_{i=1}^{n} M_{X_i}(t)$

↑ sett inn fra blått hefte

$$= \prod_{i=1}^{n} e^{\lambda_i (e^t - 1)} = \underbrace{e^{\left(\sum_{i=1}^{n} \lambda_i\right)(e^t - 1)}}$$

akkurat samme som i heftet

med $\mu = \sum_{i=1}^{t} \lambda_i$

$$\Rightarrow Y \sim \text{Poisson}\left(\mu = \sum_{i=1}^{n} \lambda_i\right)$$

OPPG. 4

samme for alle

$X_i \sim$ eksponential (λ) $\quad i = 1, \ldots, n$, uavh.

$Y = \sum X_i$

Finn $M_Y(t)$

$$M_Y(t) = \prod_{i=1}^{n} M_{X_i}(t)$$

$$= \prod_{i=1}^{n} \frac{1}{1 - t/\lambda}$$

$$= \frac{1}{(1 - t/\lambda)^n}$$

OPPG. 5

2 min

a) X: diameter bolt $\quad X \sim N(9.85,\ 0.05^2)$

Y: diameter mutter $\quad Y \sim N(10.0,\ 0.05^2)$

uavh.

$P(X \leq Y) = ?$

$P(X - Y \leq 0) = ?$

← kan settes opp med dobbelintegral ala det ingeborg gjorde i forelesning om sum av to eksponentialfordelte variabler.

b) Generelt:

$$M_{X-Y}(t) = E\left(e^{t(X-Y)}\right) = E\left(e^{tX} e^{-tY}\right)$$

$$= E(e^{tX}) \cdot E(e^{-tY})$$

$$= M_X(t) \, M_Y(-t)$$

normal:

$$M_{\bar{X}-Y}(t) = \left(e^{9.85t + (0.05)^2 t^2/2}\right) \cdot \left(e^{10.0(-t) + (0.05)^2(-t)^2/2}\right)$$

$$= e^{(9.85-10.0)t + ((0.05)^2 + (0.05)^2)t^2/2}$$

$$= e^{-0.15t + (2(0.05)^2)t^2/2}$$

μ σ^2

$$\bar{X} - \bar{Y} \sim N(-0.15, 2(0.05)^2)$$

c) $$P(\bar{X}-\bar{Y} \leq 0) = P\left(Z \leq \frac{0-(-0.15)}{\sqrt{2} \cdot 0.05}\right)$$

$$= P(Z \leq 2.12) = \underline{\underline{0.\cancel{9788}}} \quad 0.9830$$

tabell