Snødybder

Gj.snitt: 33,95

Median: 27

St.avvik: 26,72

Ls huskeregel:
ca 95% i normalfordeling

ligger innafor ±2 st.avvik

Kvartilbredde: 41

Temperatur

Gj.snitt: -1,41

Median: -1,15

St.avvik: 3,83

Kvartilbredde: 5.1

Oppgave 1

Median: 7,25

Oppgave 2

a)

$E(X) = 5$

$SD(X) = 2$

$E(Y \mid X=6) = 6$

$SD(Y \mid X=6) = 1$

Positiv samvariasjon mellom x og y (når x øker så øker også y)

$\Rightarrow$ positiv korrelasjon (og kovarians)

Oppgave 3

antall

X_i: # besøk på t_i timer

$X_1, \dots, X_n$: # besøk i n disjunkte intervall

$X_i \sim Poi(\lambda t_i)$

uavh.

$$= \frac{(\lambda t_i)^{x_i}}{x_i!} e^{-\lambda t_i}, \quad x_i = 0, 1, \dots$$

a) $\lambda = 10$, $t_1 = 1$

$$P(X_1 = 8) = \frac{10^8}{8!} e^{-10} = \underline{\underline{0.1126}}$$

$$P(X_1 \geq 8) = 1 - P(X_1 < 8)$$

$$= 1 - P(X_1 \leq 7)$$

tabell $\lambda = 10$

$$= 1 - 0.2202 = \underline{\underline{0.7798}}$$

$$P(8 \leq X_1 \leq 12)$$

$$= P(X_1 \leq 12) - P(X_1 < 8)$$

$$= P(X_1 \leq 12) - P(X_1 \leq 7)$$

tabell $\lambda = 10$

$$= 0.7916 - 0.2202 = \underline{\underline{0.5714}}$$

b)

$$\sum t_i = 1 + 2 + 5 + 1 + 5 = 14$$

$E(\tilde{\lambda}) = 2.8\lambda$

$E(\hat{\lambda}) = \lambda$

$E(\hat{\hat{\lambda}}) = \lambda$

$Var(\tilde{\lambda}) =$

$Var(\hat{\lambda}) = 0.0714\lambda$

$Var(\hat{\hat{\lambda}}) = 0.116\lambda$

$$E(\tilde{\lambda}) = E\left(\frac{1}{n} \sum X_i\right) = \frac{1}{n} \sum E(X_i)$$

$$= \frac{1}{n} \sum \lambda t_i = \frac{\lambda}{n} \sum t_i$$

$$= \frac{\lambda}{5}(1 + 2 + 5 + 1 + 5)$$

$$= \frac{\lambda}{5} \cdot 14 = 2.8\lambda$$

$$E(\hat{\hat{\lambda}}) = E\left(\frac{1}{n} \sum \frac{X_i}{t_i}\right)$$

$$= \frac{1}{n} \sum \frac{E(X_i)}{t_i} = \frac{1}{n} \sum \frac{\lambda \cancel{t_i}}{\cancel{t_i}}$$

$$= \frac{1}{n} \sum_{i=1}^{n} \lambda = \frac{n\lambda}{n} = \lambda$$

$$Var(\hat{\lambda}) = \frac{\lambda}{\sum t_i} = \frac{\lambda}{14}$$

$$\approx 0.0714\lambda$$

$$Var(\hat{\hat{\lambda}}) = \frac{1}{n^2} \sum \frac{1}{t_i^2} Var(X_i)$$

$$= \frac{1}{n^2} \sum \frac{\lambda t_i}{t_i^2} = \frac{\lambda}{n^2} \sum \frac{1}{t_i}$$

$$= \frac{\lambda}{5^2}\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{5}+\frac{1}{1}+\frac{1}{5}\right)$$

$$= \frac{\lambda}{25}\cdot 2{,}9 = 0{,}116\lambda$$

$E(\tilde{\lambda}) \neq \lambda,$

$Var(\tilde{\lambda}) < Var(\hat{\lambda})$

$\Rightarrow \tilde{\lambda}$ best

Oppgave 4

$$\left.\begin{array}{l} X \sim Bin(n,p) \\ Y \sim Bin(n,2p) \end{array}\right\} \text{uavh.}$$

a) $n = 12,\ p = 0.2$

$P(X \leq 3) = \underline{0.795}$

$P(Y \geq 4 \mid X \leq 3)$

$\overset{X,Y \text{ uavh}}{=} P(Y \geq 4)$

$= 1 - P(Y \leq 3)$

$= 1 - 0.225 = \underline{0.775}$

$P(X + Y \leq 1)$

$= P\big((X=0 \cap Y=0) \cup$
$(X=1 \cap Y=0) \cup$
$(X=0 \cap Y=1)\big)$

$= P(X=0 \cap Y=0) + P(X=1 \cap Y=0)$
$+ P(X=0 \cap Y=1)$

$\overset{\text{uavh.}}{=} P(X=0)P(Y=0) + P(X=1)P(Y=0)$
$+ P(X=0)P(Y=1)$

$P(X=0) = 0.069$

$P(Y=0) = 0.002$

$P(X=1) = P(X\leq 1) - P(X=0)$

$= 0.275 - 0.069$

$= 0.206$

$P(Y=1) = \ldots = 0.018$

$P(X+Y\leq 1)$

$= 0.069 \cdot 0.002$

$+ 0.206 \cdot 0.002$

$+ 0.069 \cdot 0.018$

$= \underline{\underline{0.001792}}$

b)

$E(\hat{p}) = E\left(\frac{X+Y}{2n}\right) = \frac{1}{2n}(E(X)+E(Y))$

$= \frac{1}{2n}(np + n\cdot 2p) = \frac{3np}{2n} = 1.5p$

$E(\tilde{p}) = \frac{1}{3n}(E(X)+E(Y)) = \frac{1}{3n}(np+2np)$

$= 3np/3n = p$

$E(p^*) = \frac{E(X)}{2n} + \frac{E(Y)}{4n}$

$= \frac{np}{2n} + \frac{2np}{4n}$

$= \frac{np+np}{2n} = \frac{2np}{2n} = p$

$Var(\hat{p}) =$ irrelevant

$Var(\tilde{p}) = Var\left(\frac{X+Y}{3n}\right)$

$= \frac{1}{(3n)^2}(Var(X)+Var(Y))$

$= \frac{1}{(3n)^2}(np(1-p) + n2p(1-2p))$

$= \frac{np - np^2 + 2np - 4np^2}{9n^2}$

$= \frac{3np - 5np^2}{9n^2} = \frac{3p-5p^2}{9n}$

$Var(p^*) = \frac{Var(X)}{(2n)^2} + \frac{Var(Y)}{(4n)^2}$

$= \frac{np(1-p)}{4n^2} + \frac{2np(1-2p)}{16n^2}$

$= \frac{p-p^2}{4n} + \frac{2p-4p^2}{16n}$

$= \frac{2p-2p^2}{8n} + \frac{p-2p^2}{8n} = \frac{3p-4p^2}{8n}$

Finne den minste:

$$\mathrm{Var}(\tilde{p}) - \mathrm{Var}(p^*)$$

$$= \frac{3p-5p^2}{9n} - \frac{3p-4p^2}{8n}$$

$$= \frac{24p-40p^2-27p+36p^2}{72n}$$

$$= \frac{-3p-4p^2}{72n} < 0 \quad \text{for } n,p>0$$

$$\mathrm{Var}(\tilde{p}) - \mathrm{Var}(p^*) < 0$$

$$\Rightarrow \mathrm{Var}(\tilde{p}) < \mathrm{Var}(p^*)$$

$$\Rightarrow \tilde{p} \text{ best}$$

Oppgave 5

X: kjøkkenvektmåling

Y: forhandlermåling

$X \sim N(\mu, \sigma=1)$

$Y \sim N(\mu, \sigma=0.5)$

$\mathrm{cov}(X,Y) = -0.2$

a) Forv.rett + lav varians

$$E(\hat{\mu}) = E(Y) = \mu$$

$$E(\tilde{\mu}) = \frac{1}{2}(E(X)+E(Y))$$

$$= \frac{1}{2}(\mu+\mu) = \mu$$

$$\mathrm{Var}(\hat{\mu}) = \mathrm{Var}(Y)$$

$$= 0.5^2 = 0.25$$

$$\mathrm{Var}(\tilde{\mu}) = \frac{1}{2^2}\mathrm{Var}(X+Y)$$

$$= \frac{1}{4}(\mathrm{Var}(X)+\mathrm{Var}(Y)+2\,\mathrm{cov}(X,Y))$$

$$= \frac{1}{4}(1^2+0.5^2+2\cdot(-0.2))$$

$$= \frac{1}{4}(1+0.25-0.4) = 0.2125$$

$$\Rightarrow \tilde{\mu} \text{ best}$$