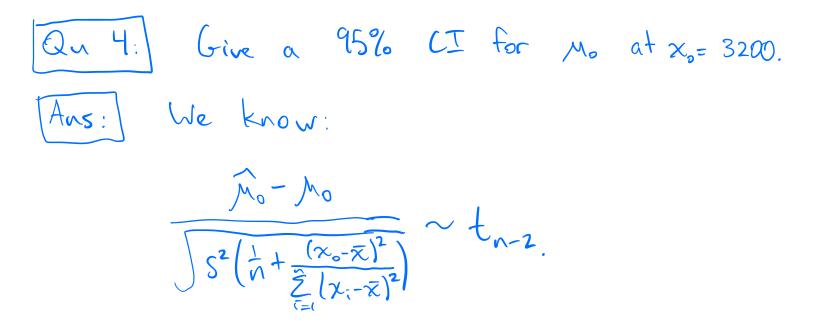
Wednesday Week 15.2 April 10th
Inference in simple linear regression
Example: Runoff revisited (based on Sering 2019 final)
Yi= B. + B. x; + Z;
Y:: runoff (mm/yr) in year i
x:: precipitation (mm/yr) in year i
E:: roise/error in year i
Assume:
$$\hat{B}_0 = -1364$$

 $\hat{B}_1 = 1.08$
 $S^2 = 156^2$.
Qual: Show that \hat{A}_0 is unbrased
[Ans:] $E[\hat{A}_0] = E[\hat{B}_0 + \hat{B}_1 x_1]$
 $= \hat{B}_0 + \hat{B}_1 x_1 J$
[Qual: Show Var($\hat{\mu}_0$) = $G^2 (\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{\bar{E}_1(\bar{X}_1 - \bar{x})^2})$
(You may assume (or $(\bar{Y}, \bar{B}_1) = 0$)



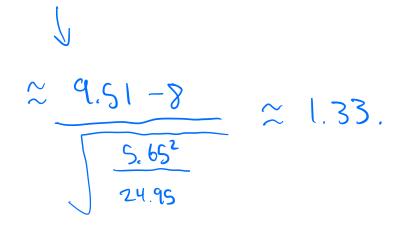
$$\widehat{\mathcal{N}}_{o} - \underbrace{t}_{.025, n-2} \int_{S}^{2} \left(\frac{1}{n} + \frac{(\chi_{o} - \overline{\chi})^{2}}{\widehat{\mathcal{Z}}_{o}(\chi_{\overline{v}} - \overline{\chi})^{2}} \right) \leq \mathcal{N}_{o} \leq \widehat{\mathcal{N}}_{o} + \underbrace{t}_{.015, n-2} \int_{S}^{2} \left(\frac{1}{n} + \frac{(\chi_{o} - \overline{\chi})^{2}}{\widehat{\mathcal{Z}}_{o}(\chi_{\overline{v}} - \overline{\chi})^{2}} \right).$$

$$\int 5^{2} \left(\frac{1}{n} + \frac{(x_{o} - \overline{x})^{2}}{\tilde{\zeta}_{i}(x_{i} - \overline{x})^{2}} \right) = \int 156^{2} \left(\frac{1}{25} + \frac{(3200 - 3200)^{2}}{\tilde{\zeta}_{i=1}^{2}(x_{i} - \overline{x})^{2}} \right)$$
$$= \int 156^{2} \left(\frac{1}{25} + \frac{0}{5} \right)$$
$$= \int \frac{156^{2}}{25} = \frac{156}{5}.$$

Also, $t_{.025, n-2} = t_{.025, 23} \approx 2.07$, and so

 $t_{.025,23} \cdot \frac{156}{5} \approx 64.58$. Thus, with 95%

would therefore not be very accurate.
[An Z:] Test if
$$\beta_1 > 8$$
 at the 95% level.
[Ans:]
We wish to test hypothesis:
Ho: $\beta_1 \leq 8$
H₁: $\beta_1 > 8$
at the 95% level ($\alpha = 0.05$).
 $\hat{\beta}_1$ is approximately Gaussian with near β_0
and variance $\frac{\sigma^2}{\frac{2}{5}(x_1-\overline{x})^2} \approx \frac{s^2}{\frac{2}{5}(x_1-\overline{x})^2}$. Hence,
Our test statistic will be:
 $Z = \frac{\hat{\beta}_1 - 8}{\int \frac{s^2}{\frac{2}{5}(x_1-\overline{x})^2}} \sim N(0, 1)$.
 $\int \frac{1}{\frac{2}{5}(x_1-\overline{x})^2} \int \frac{s}{s} \frac{1}{s} \int \frac{s^2}{s} \frac{1}{s} \int \frac{s}{s} \frac{1}{s} \int \frac{s}{s} \frac{1}{s} \frac{1}{s} \int \frac{s}{s} \frac{1}{s} \frac{1}{s} \int \frac{s}{s} \frac{1}{s} \frac{1}{s} \frac{1}{s} \int \frac{1}{s} \frac{1}{s} \frac{1}{s} \frac{1}{s} \int \frac{1}{s} \frac{1$



The critical value will be $\geq_{.05} \approx 1.64$, so there is insufficient evidence to reject the null hypothesis at the 95% level. The p-value is $I - \overline{\Psi}(1.33) \approx 0.09$.