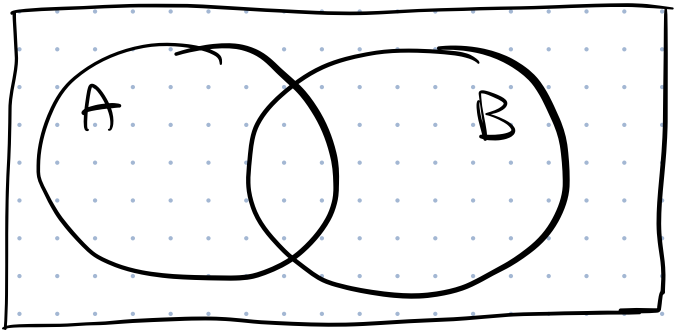
Oppgave 1, august 2022



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.5 + 0.4 - 0.3 = \underline{\underline{0.600}}$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{0.3}{0.5} = \underline{\underline{0.600}}$$

$P(C \mid A \cup B)$

Total sannsynlighet

Partisjon $\{A \cup B, (A \cup B)'\}$ av S

$$P(C) = P(C \mid A \cup B)\,P(A \cup B) +$$

$$P(C \mid (A \cup B)')\,\underbrace{P((A \cup B)')}_{= 1 - P(A \cup B)}$$

$$P(C \mid A \cup B) = \frac{P(C) - P(C \mid (A \cup B)')(1 - P(A \cup B))}{P(A \cup B)}$$

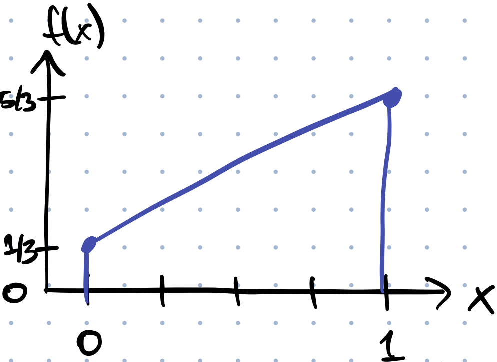
$$= \frac{0.3 - 0.2 \cdot (1 - 0{,}6)}{0{,}6} = \underline{\underline{0.367}}$$

Oppgave 1, desember 2009

a) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_0^1 k(4x+1)\,dx = k\int_0^1 (4x+1)\,dx = k\left[2x^2+x\right]_0^1$$

$$= k(2\cdot 1^2+1-0+0) = 3k = 1 \Rightarrow \underline{\underline{k=\frac{1}{3}}}$$



$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx = \int_0^x \frac{1}{3}(4x+1)\,dx$$

$$= \frac{1}{3}\left[2x^2+x\right]_0^x = \frac{1}{3}(2x^2+x-0-0) = \frac{1}{3}(2x^2+x)$$

$x \in [0,1]$

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{2x^2 + x}{3}, & x \in [0,1] \\ 1, & x > 1 \end{cases}$$

$$P\left(X > \frac{1}{4}\right) = 1 - P\left(X \leq \frac{1}{4}\right) = 1 - F\left(\frac{1}{4}\right)$$

$$= 1 - \frac{2 \cdot 0{,}25^2 + 0{,}25}{3} = \underline{\underline{0.875}}$$

$$P\left(X > \frac{1}{2} \mid X > \frac{1}{4}\right) = \frac{P(X > 1/2 \cap X > 1/4)}{P(X > 1/4)}$$

$$= \frac{P(X > 1/2)}{0.875} = \frac{1 - P(X \leq 1/2)}{0.875} = \frac{1 - F(0.5)}{0.875}$$

$$= \frac{1 - \frac{2 \cdot 0.5^2 + 0.5}{3}}{0.875} = \underline{\underline{0.762}}$$

b)

$$f(x,y) = f(x)f(y|x) = \frac{1}{3}\cancel{(4x+1)} \cdot \frac{4x+2y}{\cancel{4x+1}}$$

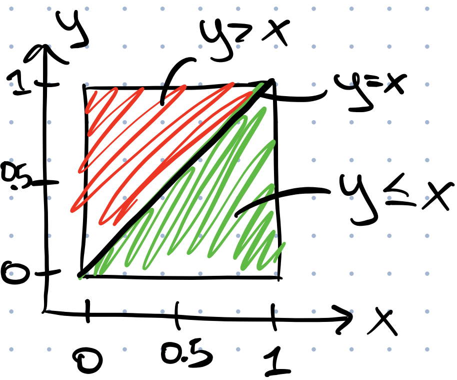
$$= \underline{\underline{\frac{4x+2y}{3}}} \quad , x,y \in [0,1] \, , \quad 0 \text{ ellers}$$

$$f(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_0^1 \frac{4x+2y}{3}\,dx$$

$$= \frac{1}{3}\left[2x^2 + 2yx\right]_0^1 = \frac{1}{3}\left(2\cdot 1^2 + 2y\cdot 1 - 0 - 0\right)$$

$$= \frac{1}{3}(2+2y) = \underline{\underline{\frac{2}{3}(y+1)}} \, , \; y \in [0,1] \, , \; 0 \text{ ellers}$$

$$P(Y \leq X)$$



$$= \int_0^1 \int_0^x f(x,y)\,dy\,dx$$

$$= \int_0^1 \int_0^x \frac{4x+2y}{3}\,dy\,dx$$

$$= \int_0^1 \frac{1}{3}\left[4xy + y^2\right]_0^x dx = \int_0^1 \frac{1}{3}(4x^2 + x^2 - 0 - 0)\, dx$$

$$= \frac{5}{3}\int_0^1 x^2 dx = \frac{5}{3}\left[\frac{1}{3}x^3\right]_0^1 = \frac{5}{9}(1^3 - 0) = \underline{\underline{\frac{5}{9}}}$$

Oppgave 2a, november 2017

X_A: # ganger en pasient oppsøker avd. etter operasjon ved A

X_B: —||— B

$X_A \sim Poi(\mu_A = 1{,}4)$, $X_B \sim Poi(\mu_B = 0{,}81)$

66 % opereres ved A

a) $P(X_A = 0) = \frac{\mu_A^0}{0!} e^{-\mu_A} = e^{-\mu_A} = e^{-1{,}4}$

$= \underline{\underline{0{,}2466}}$

$$P(X_A > 2 \mid X_A \geq 1) = \frac{P(X_A > 2 \cap X_A \geq 1)}{P(X_A \geq 1)}$$

$$= \frac{P(X_A > 2)}{P(X_A \geq 1)} = \frac{1 - P(X_A \leq 2)}{1 - P(X_A = 0)}$$

$$= \frac{1 - P(X_A = 0) - P(X_A = 1) - P(X_A = 2)}{1 - P(X_A = 0)}$$

$$= \frac{1 - 0{,}2466 - \frac{1{,}4^1}{1!}e^{-1{,}4} - \frac{1{,}4^2}{2!}e^{-1{,}4}}{1 - 0.2466}$$

$$= \underline{\underline{0{,}221}}$$

A: pasient på oppfølging kommer fra A
B: —"— B

$P(A) = 0{,}66$, $P(B) = 1 - P(A) = 0.34$

X: antall besøk på avd. for en pasient

Total sanns:

$$P(X=0) = P(X=0 \mid A)P(A) + P(X=0 \mid B)P(B)$$

$$= \frac{\mu_A^0}{0!}e^{-\mu_A}P(A) + \frac{\mu_B^0}{0!}e^{-\mu_B}P(B)$$

$$= 0{,}2466 \cdot 0{,}66 + e^{-0{,}81} \cdot 0.34 = \underline{\underline{0{,}3140}}$$

Oppgave 6, august 2019

$$X_i \sim Poi(\mu_i) = \frac{\mu_i^{x_i}}{x_i!} e^{-\mu_i}, \quad i = 1, 2, 3$$

$$Z_i = \begin{cases} 1, & X_i = X_3 = 0 \\ 0, & \text{ellers} \end{cases}, \quad i = 1, 2$$

a) Binomisk $(n=1)$ = Bernoulli

Z_i: • ett forsøk med suksess (1) eller fiasko (0)

• uavh. forsøk (har bare ett)

• lik sanns. for suksess i hvert forsøk (har bare ett):

$$p_i = P(Z_i = 1) = P(X_i = 0 \cap X_3 = 0)$$

$$= P(X_i = 0) \cdot P(X_3 = 0)$$

$$= \frac{\mu_i^0}{0!} e^{-\mu_i} \cdot \frac{\mu_3^0}{0!} e^{-\mu_3}$$

$$= e^{-\mu_i} e^{-\mu_3} = \underline{\underline{e^{-(\mu_i + \mu_3)}}}$$

$$E(Z_{+i}) = p_i = \underline{\underline{e^{-(\mu_i+\mu_3)}}}$$

$$Var(Z_{+i}) = p_i(1-p_i) = \underline{\underline{e^{-(\mu_i+\mu_3)}\left(1-e^{-(\mu_i+\mu_3)}\right)}}$$

$$cov(Z_{+1}, Z_{+2}) = E(Z_{+1}\cdot Z_{+2}) - E(Z_{+1})\,E(Z_{+2})$$

$$E(Z_{+1}Z_{+2}) = \sum_{z_1}\sum_{z_2} z_1 z_2 P(Z_{+1}=z_1, Z_{+2}=z_2)$$

$$= 0\cdot 0\cdot P(Z_{+1}=0, Z_{+2}=0)$$

$$+ 0\cdot 1\cdot \ldots + 1\cdot 0\cdot \ldots$$

$$+ 1\cdot 1\cdot P(Z_{+1}=1, Z_{+2}=1)$$

$$= P(Z_{+1}=1, Z_{+2}=1)$$

$$= P(X_1=0, X_2=0, X_3=0)$$

$$= P(X_1=0)P(X_2=0)P(X_3=0)$$

$$= e^{-\mu_1}e^{-\mu_2}e^{-\mu_3} = e^{-(\mu_1+\mu_2+\mu_3)}$$

$$\mathrm{cov}(Z_{1}, Z_{2}) = E(Z_{1}Z_{2}) - E(Z_{1})E(Z_{2})$$

$$= e^{-(\mu_1+\mu_2+\mu_3)} - e^{-(\mu_1+\mu_3)}e^{-(\mu_2+\mu_3)}$$

$$= e^{-(\mu_1+\mu_2+\mu_3)}\left(1 - e^{-\mu_3}\right)$$

Oppgave 3, desember 2009

Metode A: $X_i \sim N(\mu, \sigma_A^2)$, $i = 1, .., n$

Metode B: $Y_i \sim N(\mu, \sigma_B^2)$, $i = 1, .., m$

X_i, Y_j uavh. $\forall\ i, j$

σ_A^2 og σ_B^2 kjent, μ ukjent

$$\hat{\mu} = a\bar{X} + b\bar{Y} = a\frac{1}{n}\sum_{i=1}^{n} X_i + b\frac{1}{m}\sum_{i=1}^{m} Y_i$$

konstanter

a) $E(\hat{\mu}) = E\left(a\frac{1}{n}\sum X_i + b\frac{1}{m}\sum Y_i\right)$

$$= \frac{a}{n}\sum E(X_i) + \frac{b}{m}\sum E(Y_i)$$

$$= \frac{a}{n}\sum_{i=1}^{n}\mu + \frac{b}{m}\sum_{i=1}^{m}\mu$$

$$= \frac{a}{n}(\overbrace{\mu+\mu+\dots+\mu}^{n \text{ ganger}}) + \frac{b}{m}(\overbrace{\mu+\mu+\dots+\mu}^{m \text{ ganger}})$$

$$= \frac{a}{n} n\mu + \frac{b}{m} m\mu = a\mu + b\mu$$

$$= \underline{\underline{(a+b)\mu}}$$

$$\mathrm{Var}(\hat{\mu}) = \mathrm{Var}\left(\frac{a}{n}\sum X_i + \frac{b}{m}\sum Y_i\right)$$

X, Y uavh.

$$= \frac{a^2}{n^2}\mathrm{Var}\left(\sum X_i\right) + \frac{b^2}{m^2}\mathrm{Var}\left(\sum Y_i\right)$$

X_i uavh.
Y_i uavh.

$$= \frac{a^2}{n^2}\sum \mathrm{Var}(X_i) + \frac{b^2}{m^2}\sum \mathrm{Var}(Y_i)$$

$$= \frac{a^2}{n^2} n \cdot \sigma_A^2 + \frac{b^2}{m^2}\sigma_B^2$$

$$= \underline{\underline{\frac{a^2\sigma_A^2}{n} + \frac{b^2\sigma_B^2}{m}}}$$

b) Best estimator:
- Forventningsrett
- Lavest mulig varians

Forventningsrett

$$E(\hat{\mu}) = (a+b)\mu = \mu$$

$$\Rightarrow a+b=1 \quad \Rightarrow b = 1-a$$

Minst varians:

$$\mathrm{var}(\hat{\mu}) = \frac{a^2\sigma_A^2}{n} + \frac{\overbrace{(1-a)^2}^{=b^2}\sigma_B^2}{m}$$

$$\frac{\partial}{\partial a}\mathrm{var}(\hat{\mu}) = \frac{2a\sigma_A^2}{n} + \frac{2(1-a)(-1)\sigma_B^2}{m}$$

$$= \frac{2a\sigma_A^2}{n} - \frac{2\sigma_B^2}{m} + \frac{2a\sigma_B^2}{m} = 0$$

$$\frac{\cancel{2}a\sigma_A^2}{n} + \frac{\cancel{2}a\sigma_B^2}{m} = \frac{\cancel{2}\sigma_B^2}{m}$$

$$a(\sigma_A^2 m + \sigma_B^2 n) = \sigma_B^2 n$$

$$\underline{\underline{a = \frac{\sigma_B^2 n}{\sigma_A^2 m + \sigma_B^2 n}}} \quad \text{og} \quad b = 1-a = \underline{\underline{\frac{\sigma_A^2 m}{\sigma_A^2 m + \sigma_B^2 n}}}$$

c) $\hat{\mu}$ er lin.komb. av normalfordelte variabler ($\bar{X}_i$ og $\bar{Y}_i$), så $\hat{\mu}$ er også normalfordelt. $E(\hat{\mu})$ og $Var(\hat{\mu})$ har vi fra a):

$$\underline{\underline{\hat{\mu} \sim N\left((a+b)\mu, \frac{a^2\sigma_A^2}{n} + \frac{b^2\sigma_B^2}{m}\right)}}$$

$(1-\alpha)\cdot 100\%$ KI for μ:

1) pivotal

$$\frac{\hat{\mu} - E(\hat{\mu})}{SD(\hat{\mu})} \sim N(0,1)$$

2) kvantiler

$$P\left(\underbrace{z_{1-\frac{\alpha}{2}}}_{=-z_{\frac{\alpha}{2}}} \le \frac{\hat{\mu} - E(\hat{\mu})}{SD(\hat{\mu})} \le z_{\frac{\alpha}{2}}\right) = 1-\alpha$$

$= -z_{\frac{\alpha}{2}}$ fordi N(0,1)

3+4) løse mhp μ og sette sammen

$$-z_{\frac{\alpha}{2}} \cdot SD(\hat{\mu}) \le \hat{\mu} - (a+b)\mu$$

$$(a+b)\mu \leq \hat{\mu} + z_{\frac{\alpha}{2}} SD(\hat{\mu})$$

$$\mu \leq \frac{\hat{\mu} + z_{\frac{\alpha}{2}} SD(\hat{\mu})}{a+b}$$

$$\hat{\mu} - (a+b)\mu \leq z_{\frac{\alpha}{2}} SD(\hat{\mu})$$

$$\hat{\mu} - z_{\frac{\alpha}{2}} SD(\hat{\mu}) \leq (a+b)\mu$$

$$\frac{\hat{\mu} - z_{\frac{\alpha}{2}} SD(\hat{\mu})}{a+b} \leq \mu$$

$$P\left(\frac{\hat{\mu} - z_{\frac{\alpha}{2}} SD(\hat{\mu})}{a+b} \leq \mu \leq \frac{\hat{\mu} + z_{\frac{\alpha}{2}} SD(\hat{\mu})}{a+b} \right)$$

$$= 1 - \alpha$$

$$SD(\hat{\mu}) = \sqrt{\frac{a^2\sigma_A^2}{n} + \frac{b^2\sigma_B^2}{m}}$$

5) intervall

$$\left[\frac{\hat{\mu} - z_{\frac{\alpha}{2}} \sqrt{a^2\sigma_A^2/n + b^2\sigma_B^2/m}}{a+b} \right.,$$

$$\frac{\hat{\mu} + z_{\frac{\alpha}{2}}\sqrt{a^2\sigma_A^2/n + b^2\sigma_B^2/m}}{a+b}\Bigg]$$

d) Kortest mulig intervall

Se LF for utregning, hovedtrekk:
øvre minus nedre grense minst mulig
derivere mhp. a og b og sette lik 0
(log av diff. er enklere)