Oppgave 1

Stok. forsøk: Ett terningkast

a) $S = \{$ ⚀, ⚁, ⚂, ⚃, ⚄, ⚅ $\}$

b) - diskret tellevariabel

- (tellbart) endelig antall verdier

c) 6 verdier, én for hvert terningkast

Mest logisk:

$X(e) \in \{1, 2, 3, 4, 5, 6\}$

for $e \in S$

d) Sanns. for ⚄:

Uniform sanns. modell, så $\underline{\underline{1/6}}$

e) $P(\underbrace{X = 5}_{\text{hendelse}})$

f) 1/6 for hvert terningkast

$$\Rightarrow P(X = x) = f(x) = 1/6$$

for alle $x \in \underbrace{\{1, 2, 3, 4, 5, 6\}}_{\text{verdimengde}}$

punktsannsynlighet

Oppgave 2

a)

Må ha: $- f(x) \geq 0 \ \forall x$

$- \sum_x f(x) = 1$

Ser at $f(x) \geq 0 \ \forall x \in \{-2, -1, .., 1\}$

$$\sum_x f(x) = \sum_{x=-2}^{2} f(x) = f(-2) + f(-1) + \dots + f(2)$$

$$= 0.1 + 0.1 + 0.5 + 0.2 + 0.1$$

$$= \underline{\underline{1}}$$

$\Rightarrow f(x)$ gyldig

b) $P(X \geq 0)$

$$= P(X=0 \cup X=1 \cup X=2)$$

$$= P(X=0) + P(X=1) + P(X=2)$$

$$= 0.5 + 0.2 + 0.1 = \underline{\underline{0.8}}$$

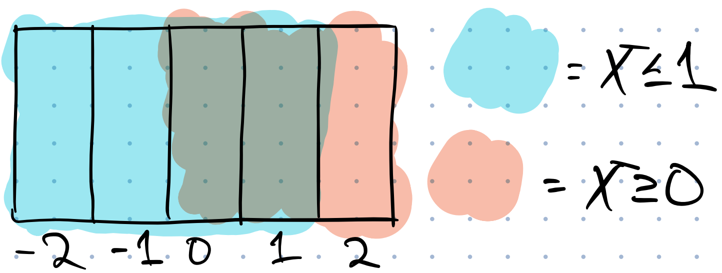
$$P(\overbrace{X \geq 0}^{A} \mid \overbrace{X \leq 1}^{B})$$

$$= \frac{P(X \geq 0 \cap X \leq 1)}{P(X \leq 1)}$$

$$= \frac{P(X=0 \cup X=1)}{P(X \leq 1)}$$

$$= \frac{P(X=0) + P(X=1)}{1 - P(X=2)} = \frac{0.5 + 0.2}{1 - 0.1}$$

$$= \frac{7}{9} \approx \underline{\underline{0.778}}$$



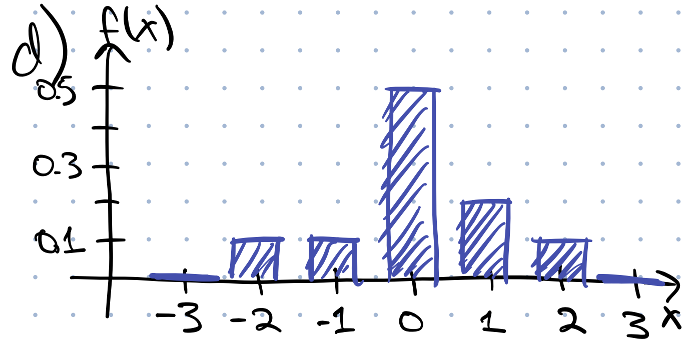
c) $F(x) = \sum_{t=0}^{x} f(t)$

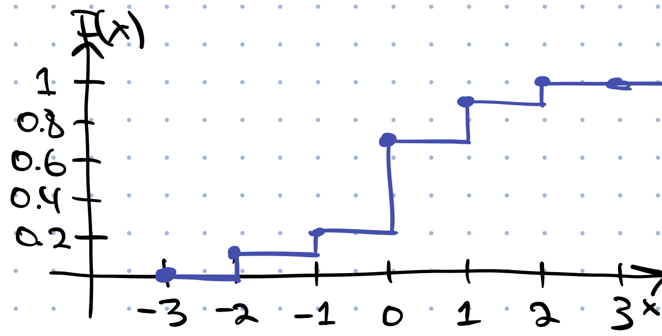
	+0.1	+0.1	+0.5	+0.2	+0.1
X	-2	-1	0	1	2
F(x)	0.1	0.2	0.7	0.9	1

0 for $x < -2$

1 for $x \geq 2$

d)





+Python

Oppgave 3

$$f(x) = \begin{cases} k(1-x^2), & -1 \le x \le 1 \\ 0, & \text{ellers} \end{cases}$$

a)

- $f(x) \ge 0 \ \forall \ x \in \mathbb{R}$

$\rightarrow$ ok for $k \ge 0$ fordi $(1-x^2) \ge 0 \ \forall \ x \in [-1,1]$

- $\int_{-\infty}^{\infty} f(x)dx = 1$

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-1}^{1} k(1-x^2)dx$$

$$= k\left(\int_{-1}^{1} 1dx - \int_{-1}^{1} x^2 dx\right)$$

$$= k\left(\left[x\right]_{-1}^{1} - \left[\frac{1}{3}x^3\right]_{-1}^{1}\right)$$

$$= k\left(\{1-(-1)\} - \{\frac{1}{3}1^3 - \frac{1}{3}\underbrace{(-1)^3}_{=-1}\}\right)$$

$$= k\left(2 - \frac{1}{3} - \frac{1}{3}\right) = \frac{4}{3}k = 1$$

$$\underline{\underline{k = 3/4}}$$

b) Slides for graf

c)

$P(X \le 0{,}6) = F(0{,}6)$

Først: finne $F(x)$

$$F(x) = \int_{-\infty}^{x} f(t)dt = \frac{3}{4}\int_{-\infty}^{x}(1-t^2)dt$$

$$= \frac{3}{4}\int_{-1}^{x}(1-t^2)dt$$

$$= \frac{3}{4}\left(\int_{-1}^{x} 1dt - \int_{-1}^{x} t^2 dt\right)$$

$$= \frac{3}{4}\left(\left[t\right]_{-1}^{x} - \left[\frac{1}{3}t^3\right]_{-1}^{x}\right)$$

$$= \frac{3}{4}\left(\{x-(-1)\} - \{\frac{1}{3}x^3 - \frac{1}{3}\underbrace{(-1)^3}_{=-1}\}\right)$$

$$= \frac{3}{4}\left(x+1-\frac{1}{3}x^3 - \frac{1}{3}\right)$$

$$= \frac{3}{4}\left(\frac{3}{3}x - \frac{1}{3}x^3 + \frac{2}{3}\right)$$

$$= \underline{\frac{3x - x^3 + 2}{4}}$$

$$P(X \le 0{,}6) = F(0{,}6) = \frac{3 \cdot 0{,}6 - 0{,}6^3 + 2}{4}$$

$$= \underline{\underline{0{,}896}}$$

$$P(X \le 0{,}8 \mid X > 0{,}6)$$

$$= \frac{P(X \le 0{,}8 \cap X > 0{,}6)}{P(X > 0{,}6)}$$

$$= \frac{P(0.6 < X \leq 0.8)}{1 - P(X \leq 0.6)}$$

$$= \frac{F(0.8) - F(0.6)}{1 - F(0.6)}$$

$$= \frac{0,972 - 0,896}{1 - 0,896} \approx \underline{\underline{0,73}}$$

Oppgave 4

	y=0	y=1	f(x)
x=1	0,1	0,2	0,3
x=2	0,3	0,15	0,45
x=3	0,15	0,1	0,25
f(y)	0,55	0,45	

Simultanfordeling

a)

$$P(X=1) = \sum_y P(X=1, Y=y)$$

$$= 0,1 + 0,2 = \underline{\underline{0,3}}$$

$$P(X > Y+1)$$

$$= P(X=2 \cap Y=0)$$

$$+ P(X=3 \cap Y=0)$$

$$+ P(X=3 \cap Y=1)$$

$$= 0,3 + 0,15 + 0,1$$

$$= \underline{\underline{0,55}}$$

b) Summere rader + kolonner

c)

$$f_{XY}(x,y) = f_X(x) f_Y(y)$$

$$0{,}1 \overset{?}{=} 0{,}3 \cdot 0{,}55 = 0{,}165$$

nei $\Rightarrow$ avhengige