Oppgave 1

kast	antall
1	
2	
3	
4	
5	
6	
total	

se tavle

La x_i, $i=1,\dots,n$ være terningkast nr. i

La y_j, $j=1,\dots,6$ være antall kast med j øyne

Gjennomsnitt:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{n}\sum_{j=1}^{6} j\cdot y_j$$

$$= \frac{1}{n}\left(1\cdot y_1 + 2\cdot y_2 + \dots + 6\cdot y_6\right) = \text{se tavle}$$

Empirisk varians:

$$\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2$$

$$= \frac{1}{n-1}\sum_{j=1}^{6} y_j\cdot(j-\bar{x})^2$$

$= \text{se tavle}$

Empirisk standardavvik:

$\sqrt{\text{empirisk varians}} =$ se tavle

Forventningsverdi:

$$\mu = E(X) = \sum_{x} x\cdot f(x)$$

$$= \sum_{x=1}^{6} x\cdot\frac{1}{6} = \frac{1}{6}\cdot 21 = \frac{7}{2} = \underline{\underline{3.5}}$$

Varians:

$$\sigma^2 = Var(X) = E((X-\mu)^2)$$

$$= E(X^2 - 2X\mu + \mu^2)$$

$$= E(X^2) - 2\mu E(X) + \mu^2$$

$$= E(X^2) - 2\mu^2 + \mu^2$$

$$= E(X^2) - \mu^2$$

$$E(X^2) = \sum_{x} x^2 f(x)$$

$$= \sum_{x=1}^{6} x^2\cdot\frac{1}{6} = \frac{1}{6}\cdot 91$$

$$= 91/6$$

$\Rightarrow Var(X) = \frac{91}{6} - \frac{7^2}{2^2} \approx \underline{\underline{2{,}9167}}$

Standardavvik: typisk avvik fra forv.verdi

$\sigma = SD(X) = \sqrt{Var(X)}$

standard deviation

$\approx \underline{\underline{1{,}7078}}$

Oppgave 2

Stok. forsøk: kaste 3 terninger

X_i: antall øyne på terning i

Z: laveste antall øyne

a) $Z = \min(X_1, X_2, X_3)$

evt. $Z_1 = \min(X_1, X_2, X_3)$

b) $P(Z_1 = 1) > P(Z_1 = 2)$

$> P(Z_1 > 3)$ osv

$P(Z_1 = 6) \neq 0$, men litt liten

c) $E(Z_1)$

$= E(\min(X_1, X_2, X_3))$

$f(z)$ og $F(z)$: også simulering

simulering

$\Rightarrow$ får bare tall, ikke analytisk uttrykk

Oppgave 3

a) $E(X) = \sum_{x=-2}^{2} x\, f(x)$

$= -2 \cdot 0{,}1 - 1 \cdot 0{,}1 + 0 \cdot 0{,}5 + 1 \cdot 0{,}2 + 2 \cdot 0{,}1 = \underline{\underline{0{,}1}}$

b) $Var(X) = E((X-\mu)^2)$

$= E(X^2) - \mu^2$

$= \sum_{x=-2}^{2} x^2 f(x) - \mu^2$

$= (-2)^2 \cdot 0{,}1 + (-1)^2 \cdot 0{,}1 + 0^2 \cdot 0{,}5 + 1^2 \cdot 0{,}2 + 2^2 \cdot 0{,}1 - 0{,}1^2 = \underline{\underline{1{,}09}}$

$SD(X) = \sqrt{1.09} \approx \underline{\underline{1.044}}$

Oppgave 4

a) $E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$

$= \int_{-1}^{1} x \cdot \frac{3}{4} \cdot (1-x^2)\,dx$

$= \frac{3}{4}\left(\int_{-1}^{1} x\,dx - \int_{-1}^{1} x^3\,dx\right)$

$= \frac{3}{4}\left(\left[\frac{1}{2}x^2\right]_{-1}^{1} - \left[\frac{1}{4}x^4\right]_{-1}^{1}\right)$

$= \frac{3}{4}\left(\left\{\frac{1}{2} - \frac{1}{2}\right\} - \left\{\frac{1}{4} - \frac{1}{4}\right\}\right)$

$= \frac{3}{4}(0 - 0) = \underline{\underline{0}}$

b) $Var(X) = E((X-\mu)^2)$

$= E((X-0)^2) = E(X^2)$

$= \int_{-\infty}^{\infty} x^2 \cdot \frac{3}{4} \cdot (1-x^2)\,dx$

$= \frac{3}{4}\int_{-1}^{1} (x^2 - x^4)\,dx$

$$= \frac{3}{4}\left[\frac{1}{3}x^3 - \frac{1}{5}x^5\right]_{-1}^{1}$$

$$= \frac{3}{4}\left(\frac{1}{3} - \frac{1}{5} - \left(-\frac{1}{3} + \frac{1}{5}\right)\right)$$

$$= \frac{3}{4}\left(\frac{2}{15} + \frac{2}{15}\right) = \frac{3}{4} \cdot \frac{4}{15}$$

$$= \underline{\underline{1/5 = 0{,}2}}$$

$$SD(X) = \sqrt{0{,}2} \approx \underline{0{,}447}$$

Oppgave 5

$$f(x) = \alpha\beta x^{\beta-1} e^{-\alpha x^\beta}, \quad x \geq 0$$

(0 ellers)

a)

$$F(x) = \int_{-\infty}^{x} f(x)\,dx = \int_{0}^{x} f(x)\,dx$$

$$= \int_0^x \alpha\beta x^{\beta-1} e^{-\alpha x^\beta}\,dx$$

Ser at

$$\frac{d}{dx} e^{-\alpha x^\beta} = -\alpha\beta x^{\beta-1} e^{-\alpha x^\beta}$$

$$\Rightarrow -\int \alpha\beta x^{\beta-1} e^{-\alpha x^\beta}\,dx$$

$$= e^{-\alpha x^\beta} + C$$

$$\Rightarrow \int_0^x \alpha\beta x^{\beta-1} e^{-\alpha x^\beta}\,dx$$

$$= \left[-e^{-\alpha x^\beta}\right]_0^x$$

$$= -e^{-\alpha x^\beta} - (-1)$$

$$= \underline{\underline{1 - e^{-\alpha x^\beta}}}$$

b)

Må finne sammenheng mellom U og X:

$u = F(x)$

$u = 1 - e^{-\alpha x^{\beta}}$

$e^{-\alpha x^{\beta}} = 1 - u$

$-\alpha x^{\beta} = \log(1-u)$

$x^{\beta} = -\frac{\log(1-u)}{\alpha}$

$$x = \left(-\frac{\log(1-u)}{\alpha}\right)^{1/\beta}$$

c) $g(x) = x^2 + 1$

Knotete (og ikke alltid mulig) analytisk, men veldig enkelt med simulering!