Oppgave 1

Hvilken fordeling?

Binomisk fordeling:

X: antall riktige svar

$n = 10$

$p = 1/5 = 0.2$

$X \sim Bin(\underset{10}{n}, \underset{0.2}{p})$

$= \binom{n}{x} p^x (1-p)^{n-x}$,

$x = 0, 1, \ldots, 10$

$P(\text{stryk}) = P(\text{færre enn } 4 \text{ rette})$

$= P(X < 4) = P(X \leq 3)$

$= P(X=0) + P(X=1)$

$+ P(X=2) + P(X=3)$

$= \binom{10}{0} 0{,}2^0 (1-0.2)^{10-0}$

$+ \binom{10}{1} 0.2^1 \cdot 0.8^9$

$+ \ldots$

$\approx 0.1074 + 0{,}2684$

$+ 0.3020 + 0.2013$

$= \underline{\underline{0.8791}}$

Oppgave 2

$X \sim Bin(n, p=0{,}9)$

$P(X \geq 10) \geq 0{,}95$

$1 - P(X < 10) \geq 0{,}95$

$1 - 0.95 \geq P(X < 10)$

$= P(X \leq 9)$

$0{,}05 \geq P(X \leq 9)$

$P(X \leq 9) \leq 0.05$

$\Rightarrow \underline{\underline{n = 13}}$

Oppgave 3

X_i: antall kast som treffer område i

Multinomisk

a) X_1: # bull's eye

X_2: # dobbel

X_3: # trippel

X_4: # bom

$n = 7$ kast

$p_1 = 0.01$

$p_2 = 0.1$

$p_3 = 0.05$

$p_4 = 0.02$

Trenger en S. variabel som beskriver "resten":

X_5: # andre kast

$$p_5 = 1 - \sum_{i=1}^{4} p_i = 0.82$$

b) $P(X_1 = 0, X_2 = 2, X_3 = 0, X_4 = 1, X_5 = \underbrace{7-2-1}_{=4})$

formel-heftet

$$= \frac{7!}{0! \cdot 2! \cdot 0! \cdot 1! \cdot 4!} \cdot 0.01^0 \cdot 0.1^2 \cdot 0.05^0 \cdot 0.02^1 \cdot 0.82^4$$

$$\approx 0.0095$$

Oppgave 4

a) X: signalet som er sendt

Y: signalet som er mottatt

$P(X=1) = 0.2$

$P(Y=1 \mid X=1) = P(Y=0 \mid X=0) = 0.9$

$P(Y=1)$

$$P(\overbrace{Y=1}^{A}) = \sum_{x=0}^{1} P(Y=1 \mid \overbrace{X=x}^{B_i}) P(X=x)$$

$$= P(Y=1 \mid X=0) P(X=0) + P(Y=1 \mid X=1) P(X=1)$$

$$P(Y=1 \mid X=0) = 1 - P(Y=0 \mid X=0) = 1 - 0.9 = 0.1$$

$$P(X=0) = 1 - P(X=1) = 1 - 0.2 = 0.8$$

$$P(Y=1) = 0.1 \cdot 0.8 + 0.9 \cdot 0.2 = 0.26$$

$$P(\underbrace{X=1}_{A} \mid \underbrace{Y=1}_{B})$$

$$P(X=1 \mid Y=1) = \frac{P(Y=1 \mid X=1) P(X=1)}{P(Y=1)}$$

$$= \frac{0.9 \cdot 0.2}{0.26} \approx \underline{\underline{0,692}}$$

b) X_i: signal sendt i pos. i

Y_i: signal mottatt i pos. i

$i = 1, \ldots, 5$

$P(Y_i = 1 \mid X_i = 1)$

$= P(Y_i = 0 \mid X_i = 0) = 0.9 \quad \forall i$

Y_i uavh. av X_j og Y_j

P(korrekt mottatt i minst 4 av 5 posisjoner)

Først:

V: antall korrekt mottatt av $n=5$

Hvorfor binomisk + $P(V \geq 4)$; 2 min

- korrekt / galt mottatt
- n uavh. forsøk

- Sanns. for korrekt mottatt $= p$ i hvert forsøk $= 0.9$

$\Rightarrow V \sim \mathrm{Bin}(n=5, p=0.9)$

$$P(V \geq 4) = P(V=4) + P(V=5)$$

$$= \binom{5}{4} 0{,}9^4\, 0.1^1 + \binom{5}{5} 0.9^5\, 0.1^0$$

$$\approx 0.3281 + 0{,}5905 \approx \underline{\underline{0{,}919}}$$

ELLER: tabell

$P(V \geq 4) = 1 - P(V < 4)$

$= 1 - P(V \leq 3)$

$= 1 - 0{,}081$

$= \underline{\underline{0{,}919}}$

La $X = (X_1, X_2, X_3, X_4, X_5)$ og tilsvarende for Y.

$$P(X = \underbrace{(0,0,0,0,0)}_{0} \mid Y = \underbrace{(0,0,0,0,0)}_{0}) = ?$$

$$P(X=0 \mid Y=0) = \frac{P(Y=0 \mid X=0) P(X=0)}{P(Y=0)}$$

$$P(Y=0 \mid X=0) = P(X_1 = 0 \mid Y_1 = 0) \cdot$$

$(0,0,0,0,0)$

$$P(X_2 = 0 \mid Y_2 = 0) \cdot \ldots = 0{,}9^5$$

$$P(X=0) = P(X=1) = 0{,}35$$

$(0,0,0,0,0)$

$$P(Y=0) = \sum_x P(Y=0 \mid X=x) P(X=x)$$

$$= P(Y=0 \mid X=0) P(X=0)$$

$$+ \underbrace{P(Y=0 \mid X=1)}_{=0{,}1^5} P(X=1)$$

$$+\sum_{x} P(Y=0 \mid X=x)\underbrace{P(X=x)}_{=0.01}$$

$x \neq (0,0,0,0,0)$
$x \neq (1,1,1,1,1)$

$P(Y=0 \mid X=x)$:

#1ere $\sim \text{Bin}(5, 0.1)$

1: $\binom{5}{1} 0.1^1 \cdot 0.9^4$

00001
00010
00100
01000
10000

2: $\binom{5}{2} 0.1^2 \cdot 0.9^3$

osv

$\Rightarrow P(Y=0)$

$= 0.9^5 \cdot 0.35$

$+ 0.1^5 \cdot 0.35$

$+ (5 \cdot 0.1^1 \cdot 0.9^4$

$+ 10 \cdot 0.1^2 \cdot 0.9^3$

$+ 10 \cdot 0.1^3 \cdot 0.9^2$

$+ 5 \cdot 0.1^4 \cdot 0.9) \cdot 0.01$

≈ 0.2108

$$\Rightarrow P(X=0 \mid Y=0) \approx \frac{0.9^5 \cdot 0.35}{0.2108}$$

$\approx \underline{\underline{0.98}}$

Oppgave 5

X: antall med brudd
inv. i elspark.ulykke

$X \sim \text{Bin}(n, p)$

a) $n = 17$, $p = 0.2$

i) $P(X=4) = \binom{17}{4} 0.2^4 \cdot 0.8^{13}$

$\approx \underline{0.209}$

ii) $P(X \geq 4) = 1 - P(X \leq 3)$

$= 1 - 0.549 = \underline{\underline{0.451}}$

iii) $P(\overbrace{X \geq 6}^{A} \mid \overbrace{X \geq 4}^{B})$

$$= \frac{P(X \geq 6 \cap X \geq 4)}{P(X \geq 4)}$$

$$= \frac{P(X \geq 6)}{P(X \geq 4)}$$

$$= \frac{1 - P(X \leq 5)}{1 - P(X \leq 3)} = \frac{1 - 0.894}{1 - 0.549}$$

$\approx \underline{\underline{0.235}}$