Oppgave 1

$X \sim N(0, \sigma = 2)$

$Y \sim N(1, \sigma = 1)$

X og Y er uavh.

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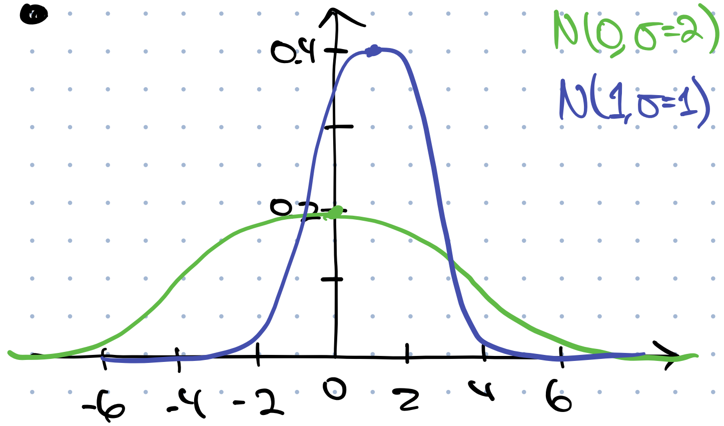

$$f_X(0) = \frac{1}{\sqrt{2\pi}\cdot 2} \exp\left\{-\frac{1}{2}\frac{(0-0)^2}{2^2}\right\}$$

$$\approx 0.2$$

$$f_Y(0) = \frac{1}{\sqrt{2\pi}\cdot 1} \exp\left\{-\frac{1}{2}\frac{(1-1)^2}{1^2}\right\}$$

$$\approx 0.4$$

• $$P(X \leq 1) = P\left(\frac{X-0}{2} \leq \frac{1-0}{2}\right)$$

$$= P(Z \leq 0.5)$$

← tabell

$$= \underline{\underline{0.6915}}$$

$$P(Y \geq -1) = P\left(\frac{Y-1}{1} \geq \frac{-1-1}{1}\right)$$

$$= P(Z \geq -2) = 1 - P(Z \leq -2)$$

$$= 1 - 0.0228 = \underline{\underline{0.9772}}$$

$X - Y$: $a_1 = 1$, $a_2 = -1$

$b = 0$

$$X - Y \sim N\left(\overset{1}{a_1}\overset{0}{\mu_X} + \overset{-1}{a_2}\overset{1}{\mu_Y} + \overset{0}{b},\ \sqrt{\underset{1}{a_1^2}\underset{4}{\sigma_X^2} + \underset{1}{a_2^2}\underset{1}{\sigma_Y^2}}\right)$$

$$= N(-1, \sqrt{5})$$

$$P(X - Y \leq 0)$$

$$= P\left(\frac{(X-Y)+1}{\sqrt{5}} \leq \frac{0+1}{\sqrt{5}}\right)$$

$$= P(Z \leq 0.447) = \underline{0.6736}$$

Oppgave 2

X: vekt hodekål (kg)

$X \sim N(2, \sigma = 0.5)$

a)

1) $P(X \leq 1.5) \approx \underline{\underline{0.1587}}$

2) $P(2 \leq X \leq 2.5)$

$= P(X \leq 2.5) - P(X \leq 2)$

$\approx \underline{\underline{0.3413}}$

$P(2 \leq X \leq 2.5 \mid X \geq 1.5)$

$$= \frac{P(2 \leq X \leq 2.5 \cap X \geq 1.5)}{P(X \geq 1.5)}$$

$$= \frac{P(2 \leq X \leq 2.5)}{1 - P(X \leq 1.5)} \approx \underline{\underline{0.4057}}$$

b) Klasse 1: veier minst 1.5 kg

c) $P(X > m) = 0.05$

$m = ?$

$$P\left(\frac{X-2}{0.5} > \frac{m-2}{0.5}\right)$$

$$= P\left(Z > \frac{m-2}{0.5}\right) = 0.05$$

$\alpha = 0.05$

$$\frac{m-2}{0.5} = z_{0.05} = 1.645$$

$m = 1.645 \cdot 0.5 + 2$

$= \underline{\underline{2.8225}}$

Oppgave 3

X: tid til pasient sovner etter medisininntak

$X \sim N(\mu, \sigma)$

a) $X \sim N(35, \sigma = 10)$

$P(X \leq 20)$

$= P\left(\frac{X-35}{10} \leq \frac{20-35}{10}\right)$

$= P(Z \leq -1.5) \overset{\text{tabell}}{=} \underline{\underline{0.0668}}$

$P(X > 40 \mid X > 20)$

$= \frac{P(X > 40 \cap X > 20)}{P(X > 20)}$

$= \frac{P(X > 40)}{P(X > 20)} = \frac{1 - P(X \leq 40)}{1 - P(X \leq 20)}$

$= \frac{1 - P\left(Z \leq \frac{40-35}{10}\right)}{1 - P(Z \leq -1.5)}$

$= \frac{1 - P(Z \leq 0.5)}{1 - P(Z \leq -1.5)} \overset{\text{tabell}}{=} \frac{1 - 0.6915}{1 - 0.0668}$

$\approx \underline{\underline{0.3306}}$

X_1: tid til pasient 1 sovner

X_2: tid til pasient 2 sovner

X_1 og X_2 uavh.

$P(X_1 + X_2 > 90)$

$a_1 = 1,\ a_2 = 1,\ b = 0$

$\mu_1 = \mu_2 = 35,\ \sigma_1 = \sigma_2 = 10$

$X_1 + X_2 \sim N(35+35, \sqrt{10^2 + 10^2})$

$= N(70, \sqrt{2} \cdot 10)$

$P(X_1 + X_2 > 90)$

$= P\left(\frac{(X_1+X_2) - 70}{\sqrt{2} \cdot 10} > \frac{90-70}{\sqrt{2} \cdot 10}\right)$

$= P\left(Z > \frac{20}{\sqrt{2} \cdot 10}\right) = P(Z > \sqrt{2})$

$= 1 - P(Z \leq 1.414)$

$\overset{\text{tabell}}{=} 1 - 0.9207 = \underline{\underline{0.0793}}$

Oppgave 4

$X \sim N(1, \sigma_X = 1)$

$Y \sim N(0, \sigma_Y = 1)$

X og Y uavh.

a)

$P(2X > 3) = P(X > 3/2)$

$= 1 - P(X \leq 1.5)$

$= 1 - P\left(\frac{X-1}{1} \leq \frac{1.5-1}{1}\right)$

$= 1 - P(Z \leq 0.5)$

$= 1 - 0.6915$

$= \underline{\underline{0.3085}}$

$P(2X > 3 \mid Y > 0)$

$= \frac{P(2X > 3 \cap Y > 0)}{P(Y > 0)}$

uavh.

$= \frac{P(2X>3)\cancel{P(Y>0)}}{\cancel{P(Y>0)}}$

$= \underline{\underline{0.3085}}$

$X - Y$ lin.komb. av normalfordelte s.v.

$\Rightarrow X - Y$ også normal

$X - Y \sim N(1, \sqrt{1^2 + 1^2})$

$= N(1, \sqrt{2})$

$P(-1 \leq X - Y \leq 1)$

$= P\left(\frac{-1-1}{\sqrt{2}} \leq Z \leq \frac{1-1}{\sqrt{2}}\right)$

$= P(-\sqrt{2} \leq Z \leq 0)$

$= P(Z \leq 0) - P(Z \leq -1.414)$

$= 0.5 - 0.0793 = \underline{0.4207}$