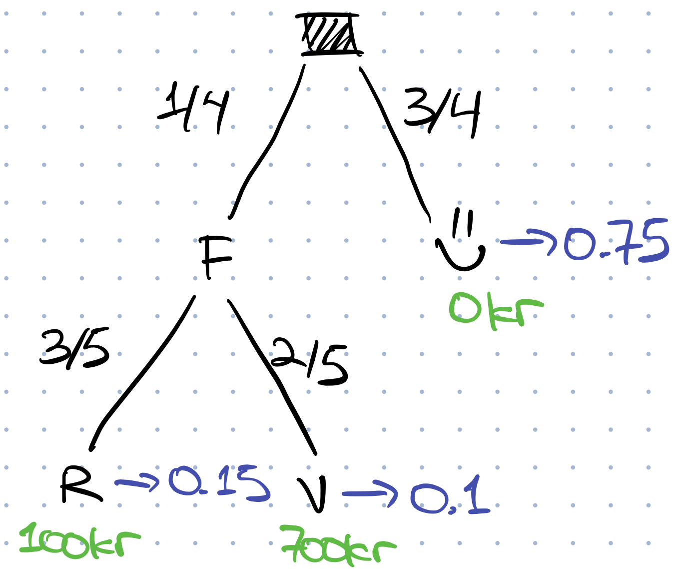
Oppgave 1



a) X: tap på 1 komponent

x	0	100	700
$P(X=x)$	0,75	0,15	0,1

$$E(X) = \sum_x xP(X=x)$$

$$= 0 \cdot 0.75 + 100 \cdot 0,15 + 700 \cdot 0,1$$

$$= \underline{\underline{85}}$$

$$Var(X) = \sum_x x^2 P(X=x) - 85^2$$

$$= 0^2 \cdot 0.75 + 100^2 \cdot 0,15 + 700^2 \cdot 0.1 - 85^2$$

$$= \underline{\underline{43275}}$$

b) Y: totalt tap på 100 enheter

$$Y = \sum_{i=1}^{100} X_i$$

← tap enhet i

$E(X_i) = 85 = \mu$

$Var(X_i) = 43275 = \sigma^2$

Antar: komponentene har feil uavh. av hverandre

$\Rightarrow X_i$ uavh.

$n = 100 > 30$

→ sentralgrenseteoremet!

SGT: $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \approx N(0,1)$ tilnærmet fordelt

$\Rightarrow \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \approx N(\mu, \sigma/\sqrt{n})$

$\Rightarrow \sum_{i=1}^{n} X_i \approx N(n\cdot\mu, \sqrt{n}\cdot\sigma)$

$Y = \sum_{i=1}^{100} X_i \approx N\left(100\cdot 85,\ 10\cdot\sqrt{43275}\right)$

$P(Y > 10000)$

standardiserer

$= P\left(Z_1 > \frac{10000-8500}{10\cdot\sqrt{43275}}\right)$

$\approx P(Z_1 > 0.721)$

$= 1 - P(Z_1 \leq 0.721)$

$= 1 - 0.7642$

$= \underline{\underline{0.2358}}$

Regneregler for $E(\bar{X})$ og $Var(\bar{X})$

$X_i \sim f(x) \ \forall i$

SGT:

$Z_1 = \frac{\bar{X} - E(X_i)}{\sqrt{Var(X_i)/n}} \approx N(0,1)$

$\Rightarrow \bar{X} \approx N\left(E(X_i), \sqrt{Var(X_i)/n}\right)$

$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$

$\Rightarrow \sum_{i=1}^{n} X_i \approx N\left(nE(X_i), \sqrt{n\, Var(X_i)}\right)$

Obs! Hvis $f(x) = N(\mu, \sigma)$, blir alt eksakt normalfordelt!

$n\cdot X$ vs $\sum X_i$, der $X, X_i \sim f(x)$ uavh. $\forall i$

$E(nX) = n\cdot E(X)$

$E\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} E(X_i)$

$= n\cdot E(X)$

likt

$Var(nX) = n^2 Var(X)$

$Var\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} Var(X_i)$

$= n\cdot Var(X)$

ikke likt!

Oppgave 2

X: antall laks med lus av $500 = n$

a)
- vanh. fisk (plukket tilfeldig)
- $n=500$ forsøk
- 2 utfall
- $P(\text{lus})=0{,}12$ i alle forsøk

$\Rightarrow$ Binomisk,

$$X \sim Bin(500, 0{,}12)$$

b) $E(X) = np = 500 \cdot 0.12 = \underline{\underline{60}}$

c) $P(X \leq 50)$

Sjekk: $np = 60 > 5$

$n \cdot (1-p) = 500 \cdot 0.88 = 440 > 5$

$\Rightarrow$ normaltilnærming ok!

$$X \approx N(60, \sqrt{500 \cdot 0{,}12 \cdot 0{,}88}) = N(60, \sqrt{52.8})$$

$$P(X \leq 50) = P\left(Z \leq \frac{50-60}{\sqrt{52.8}}\right)$$

$$= P(Z \leq -1{,}376)$$

$$= \underline{\underline{0{,}0838}}$$

Kontinuitetskorreksjon:

$$P(X \leq 50 + 0{,}5)$$

$$= P\left(Z \leq \frac{50.5-60}{\sqrt{52.8}}\right)$$

$$= P(Z \leq -1{,}307)$$

$$= \underline{\underline{0.0951}}$$

Fasit python:

$\underline{\underline{0{,}0932}}$

Oppgave 3

X_i: måling meterstokk, $i=1,..,5$

Y: lasermåling

begge i cm

$X_i \sim N(\mu, \sigma=1)$

$Y \sim N(\mu, \sigma=0.2)$

uavh

a) Først:

$\hat{\mu}_3 = \frac{1}{2}(\hat{\mu}_1 + \hat{\mu}_2)$

$\hat{\mu}_4 = \frac{1}{6}(\hat{\mu}_1 + 5\hat{\mu}_2)$

$\hat{\mu}_5 = 5\hat{\mu}_1 - 5\hat{\mu}_2$

X_i, Y uavh.

$\Rightarrow \hat{\mu}_1$ og $\hat{\mu}_2$ uavh.

$E(\hat{\mu}_1) = E(Y) = \mu$

$E(\hat{\mu}_2) = E\left(\frac{1}{5}\sum_{i=1}^{5} X_i\right) = \frac{1}{5}\sum_{i=1}^{5} E(X_i)$

$= \frac{1}{5} \cdot 5\mu = \mu$

$E(\hat{\mu}_3) = E\left(\frac{1}{2}(\hat{\mu}_1 + \hat{\mu}_2)\right)$

$= \frac{1}{2}(E(\hat{\mu}_1) + E(\hat{\mu}_2)) = \frac{1}{2}(\mu + \mu) = \mu$

$E(\hat{\mu}_4) = \frac{1}{6}(\mu + 5\mu) = \frac{6\mu}{6} = \mu$

$E(\hat{\mu}_5) = 5\mu - 5\mu = 0$

$\Rightarrow 1, 2, 3, 4$ ok!!

b) $Var(\hat{\mu}_1) = Var(Y)$

$= 0.2^2 = 0.04$

$Var(\hat{\mu}_2) = Var\left(\frac{1}{5}\sum_{i=1}^{5} X_i\right)$

$= \frac{1}{5^2}\sum Var(X_i) = \frac{1}{5^2} 5 \cdot 1$

$= 0.2$

$Var(\hat{\mu}_3) = Var\left(\frac{1}{2}(\hat{\mu}_1 + \hat{\mu}_2)\right)$

$\hat{\mu}_1, \hat{\mu}_2$ uavh

$= \frac{1}{2^2}(Var(\hat{\mu}_1) + Var(\hat{\mu}_2))$

$= \frac{1}{4}(0.04 + 0.2) = 0.06$

$Var(\hat{\mu}_4) = \frac{1}{6^2}(Var(\hat{\mu}_1) + Var(5\hat{\mu}_2))$

$= \frac{1}{6^2}(0.04 + 5^2 \cdot 0.2) = 0.14$

$Var(\hat{\mu}_5) = 5^2 Var(\hat{\mu}_1) + 5^2 Var(\hat{\mu}_2)$

$= 5^2(0.04 + 0.2) = 6$

$\Rightarrow \underset{\overset{\shortparallel}{1}}{1}, 3, 4, 2, \underset{\overset{\shortparallel}{n}}{5}$

c)

$\Rightarrow \underline{\underline{\hat{\mu}_1}}$