Forelesning 17.02.2025

Tema: Momentgenerererende funksjoner og sentralgrenseteoremet

Karina Lilleborge (vikar for Håkon Tjelmeland)
Stipendiat ved Institutt for matematiske fag

Hvordan beskriver man en stokastisk variabel?

- Diskret stokastisk variabel X:
  - Punktsannsyligheten $f_X(x) = P(X = x)$
  - Kumulativ fordelingsfunksjon

$$F_X(x) = P(X \leq x) = \sum_{t=-\infty}^{x} P(X = t)$$

- Kontinuerlig stokastisk variabel X:
  - Sannsynlighetstettheten $f_X(x)$ (OBS: $\neq P(X = x)$)
  - Kumulativ fordelingsfunksjon

$$F_X(x) = P(X \leq x) = \int_{-\infty}^{x} f_X(t)dt$$

Momenter og momentgenererende funksjon

Fra introvideoene:

Momenter og momentgenererende funksjoner

Definisjon (r-te ordens moment)

La X være en stokastisk variabel. Da er r-te ordens moment til X gitt som

$$\mu_r = E[X^r]$$

for $r \in \{1, 2, \ldots\}$.

- Merk:
  - $\mu_1 = E[X] = \mu$
  - siden $\text{Var}[X] = E[X^2] - E[X]^2$ har vi at $\mu_2 = E[X^2] = \sigma^2 - \mu_1^2$

Definisjon (Momentgenererende funksjon)

Momentgenererende funksjon til en stokastisk variabel X er

$$M_X(t) = E[e^{tX}].$$

Teorem

La X være en stokastisk variabel med momentgenererende funksjon $M_X(t)$. For $r \in \{1, 2, \ldots\}$ har vi da at

$$M_X^{(r)}(0) = E[X^r] = \mu_r$$

Eksempel

- La $X \sim$ Poisson(λ).
  $$f_X(x) = \frac{\lambda^x}{x!}e^{-\lambda}, x = 0,1,2,...$$

1. Bestem momentgenererende funksjon for X.

Poissonfordeling, s. 25 i Blå bok

Poissonfordeling

$$f(x) = p(x;\mu) = \frac{\mu^x}{x!}e^{-\mu}\ , \quad x = 0, 1, 2, \ldots .$$

$$\mathrm{E}(X) = \mu\ , \quad \mathrm{Var}(X) = \mu\ , \quad \boxed{M_X(t) = e^{\mu(e^t-1)}.}$$

Eksempel

- La $X \sim$ Poisson(λ).
  $f_X(x) = \frac{\lambda^x}{x!} e^{-\lambda}, x = 0,1,2,...$

1. Bestem momentgenererende funksjon for X.

2. Bruk momentgenererende funksjon til å bestemme $E[X]$ og Var$[X]$.
   Hint: Var$[X] = E[X^2] - E[X]^2$

Poissonfordeling, s. 25 i Blå bok

Poissonfordeling

$$f(x) = p(x;\mu) = \frac{\mu^x}{x!}e^{-\mu}\ , \quad x = 0, 1, 2, \ldots .$$

$$\boxed{\mathrm{E}(X) = \mu\ , \quad \mathrm{Var}(X) = \mu\ ,} \quad M_X(t) = e^{\mu(e^t-1)}.$$

Regneregler for momentgenererende funksjon

Teorem

La X være en stokastisk variabel og la a være en konstant. Da har vi at

- $\star$ $M_{X+a}(t) = e^{at} M_X(t)$,
- $\star$ $M_{aX}(t) = M_X(at)$.

La $X_1, X_2, \ldots, X_n$ være uavhengige stokastiske variabler. Da har vi at

- $\star$ $M_{X_1+X_2+\ldots+X_n}(t) = M_{X_1}(t) \cdot M_{X_2}(t) \cdot \ldots \cdot M_{X_n}(t)$.

Tips til innlevering 3, oppgave 1b

Se introvideoen(!), der er det også bevis for disse regnereglene (hvis man har sett et bevis blir det lettere å "huske" (forstå) hvorfor det må stemme :-))

Sentralgrenseteoremet

Fra introvideo:

Teorem (Sentralgranseteoremet)

La $X_1, X_2, \ldots, X_n$ være uavhengige og identisk fordelte variabler med forventningsverdi $E[X_i] = \mu$ og varians $Var[X_i] = \sigma^2$. La

$$Z = \frac{\overline{X} - \mu}{\sqrt{\frac{\sigma^2}{n}}},$$

der $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ er gjennomsnittet av X_i-ene. Da vil sannsynlighetsfordelingen til Z konvergere mot en standard normalfordeling når $n \to \infty$.

Fra blå bok:

Sentralgrenseteoremet

La $X_1, X_2, \ldots X_n$ være en følge av uavhengige identisk fordelte stokastiske variabler med $\mathrm{E}(X_i) = \mu$ og $0 < \mathrm{Var}(X_i) = \sigma^2 < \infty$. La $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. Da vil for store n

$$\frac{\bar{X} - \mathrm{E}(\bar{X})}{\sqrt{\mathrm{Var}(\bar{X})}} = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

være tilnærmet standard normalfordelt.

Mer presist:

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \xrightarrow{d} Z,$$

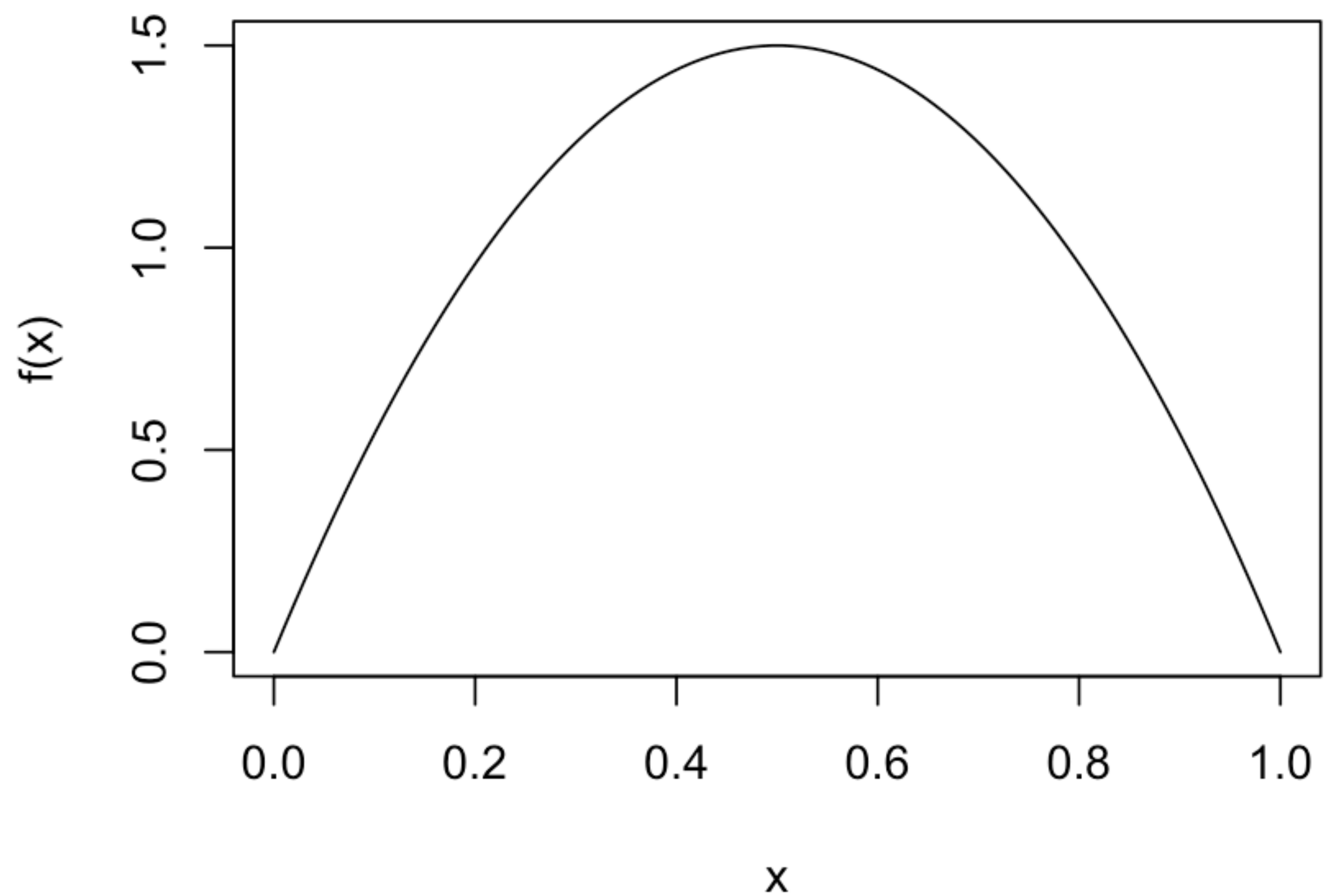
der Z er standard normalfordelt.

Eksempel

- Anta $X_1, X_2, \ldots, X_n$ er uavhengige og identisk fordelte.

$$f(x) = 6x(1-x) \quad x \in [0,1]$$

- Merk: $\int_0^1 6x(1-x)dx = 1$ (sjekk selv)
- La $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$.
- Bestem $P(\bar{X} \geq 0.55)$.



Standard normalfordeling

$$\Phi(z) = P(Z \leq z)$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999

Standard normalfordeling

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3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
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3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
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2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999

Standard normalfordeling

$$\Phi(z) = P(Z \leq z)$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
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3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
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3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999

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3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999

Eksempel (del 2)

- Finn (tilnærmet)

$$P\left(\frac{1}{2} - \frac{r}{\sqrt{20n}} \leq \bar{X} \leq \frac{1}{2} + \frac{r}{\sqrt{20n}}\right)$$

for en konstant r. X_i $i = 1,2,...,n$ har fortsatt samme fordeling som i forrige eksempel.

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for en konstant r. X_i $i = 1,2,...,n$ har fortsatt samme fordeling som i forrige eksempel.

- Mer generelt:

$$P\left(\mu - r\sqrt{\frac{\sigma^2}{n}} \leq \bar{X} \leq \mu + r\sqrt{\frac{\sigma^2}{n}}\right) \approx \Phi(r) - \Phi(-r)$$

der $\mu = E[X_i] < \infty$ og $\sigma^2 = \text{Var}[X_i] < \infty$. ($X_i$ kan ha en hvilken som helst annen fordeling så lenge dette er oppfylt)

Eksempel

Sum av uavhengige Poissonfordelte tilfeldige variabler

- La $X_1, X_2, \ldots, X_n$ være uavhengige og $X_i \sim$ Poisson(λ).
- La $\bar{X} = \dfrac{1}{n}\displaystyle\sum_{i=1}^{n} X_i$ og finn $P\left(\bar{X} \geq \dfrac{37}{40}\right)$.

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  - Hva er fordelingen til $\bar{X}$?
  - Hva er fordelingen til $Y = n\bar{X} = n\dfrac{1}{n}\displaystyle\sum_{i=1}^{n} X_i = \sum_{i=1}^{n} X_i$?

Eksempel

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- La $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ og finn $P\left(\bar{X} \geq \frac{37}{40}\right)$.
- Hva er fordelingen til $Y = n\bar{X} = n\frac{1}{n}\sum_{i=1}^{n} X_i = \sum_{i=1}^{n} X_i$?
  - $Y \sim$ Poisson$(n\lambda)$

Eksempel

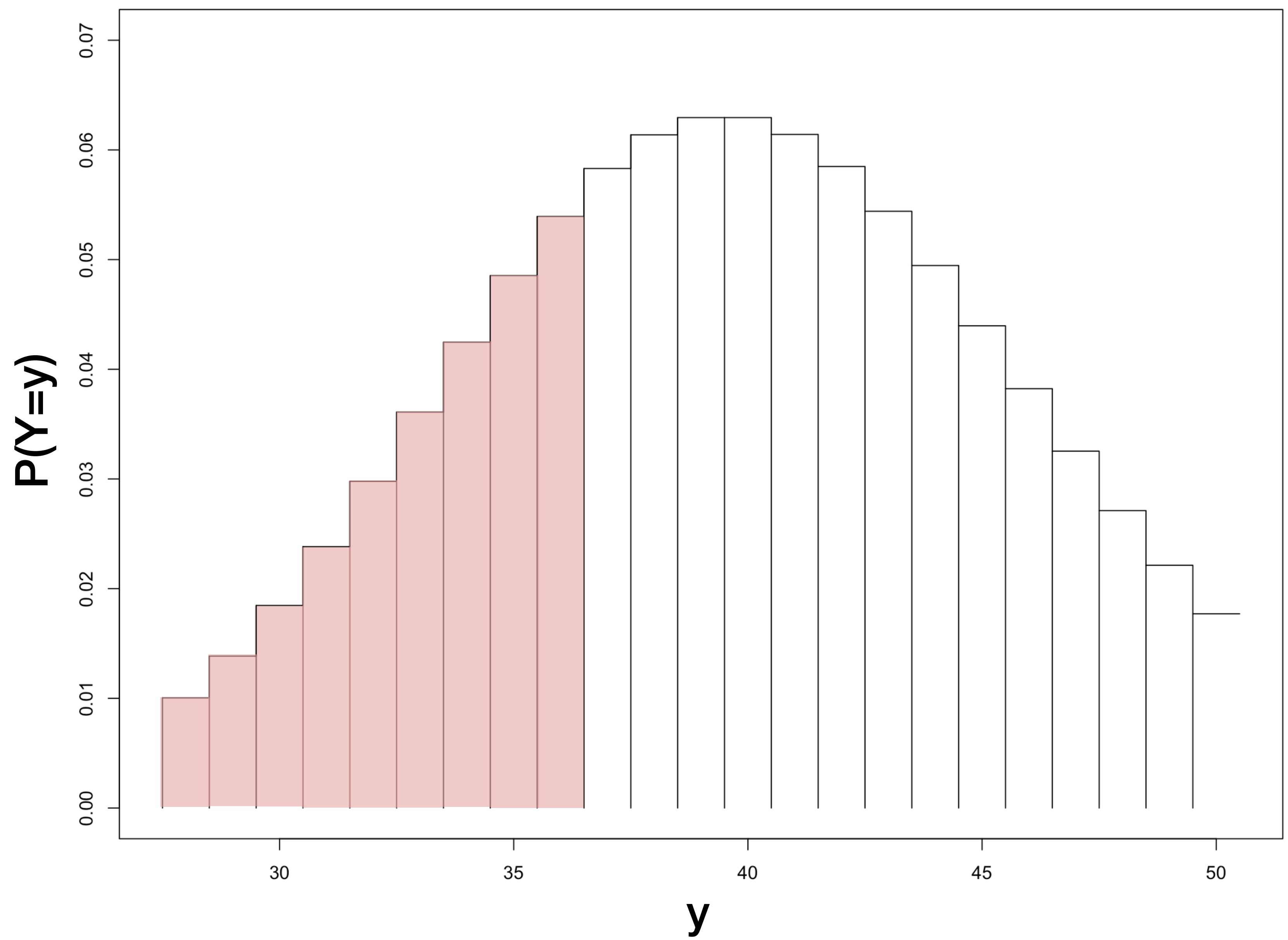
Sum av uavhengige Poissonfordelte tilfeldige variabler

- La $X_1, X_2, \ldots, X_n$ være uavhengige og $X_i \sim$ Poisson(λ). La $\lambda = 1.0$ og $n = 40$.
- La $\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$ og finn $P\left(\bar{X} \geq \frac{37}{40}\right)$.
- Hva er fordelingen til $Y = n\bar{X} = n\frac{1}{n} \sum_{i=1}^{n} X_i = \sum_{i=1}^{n} X_i$?
    - $Y \sim$ Poisson$(n\lambda)$
- Vi kan regne eksakt siden vi kjenner fordelingen til Y. (Tabell i blå bok har bare opp til $\mu = 18$, men Python har bibliotek som har funksjoner som kan regne ut for høyere parameterverdier.)
- Vi kan også finne sannsynligheten approksimativt, v.hj.a. sentralgrenseteoremet!

Kontinuitetskorreksjon

- Poissonfordelingen er diskret, og når vi vil bestemme $P(Y \leq 36)$ ønsker vi å bestemme summen av de skraverte søylene:

$$P(Y \leq 36) = \sum_{y=0}^{36} P(Y = y)$$



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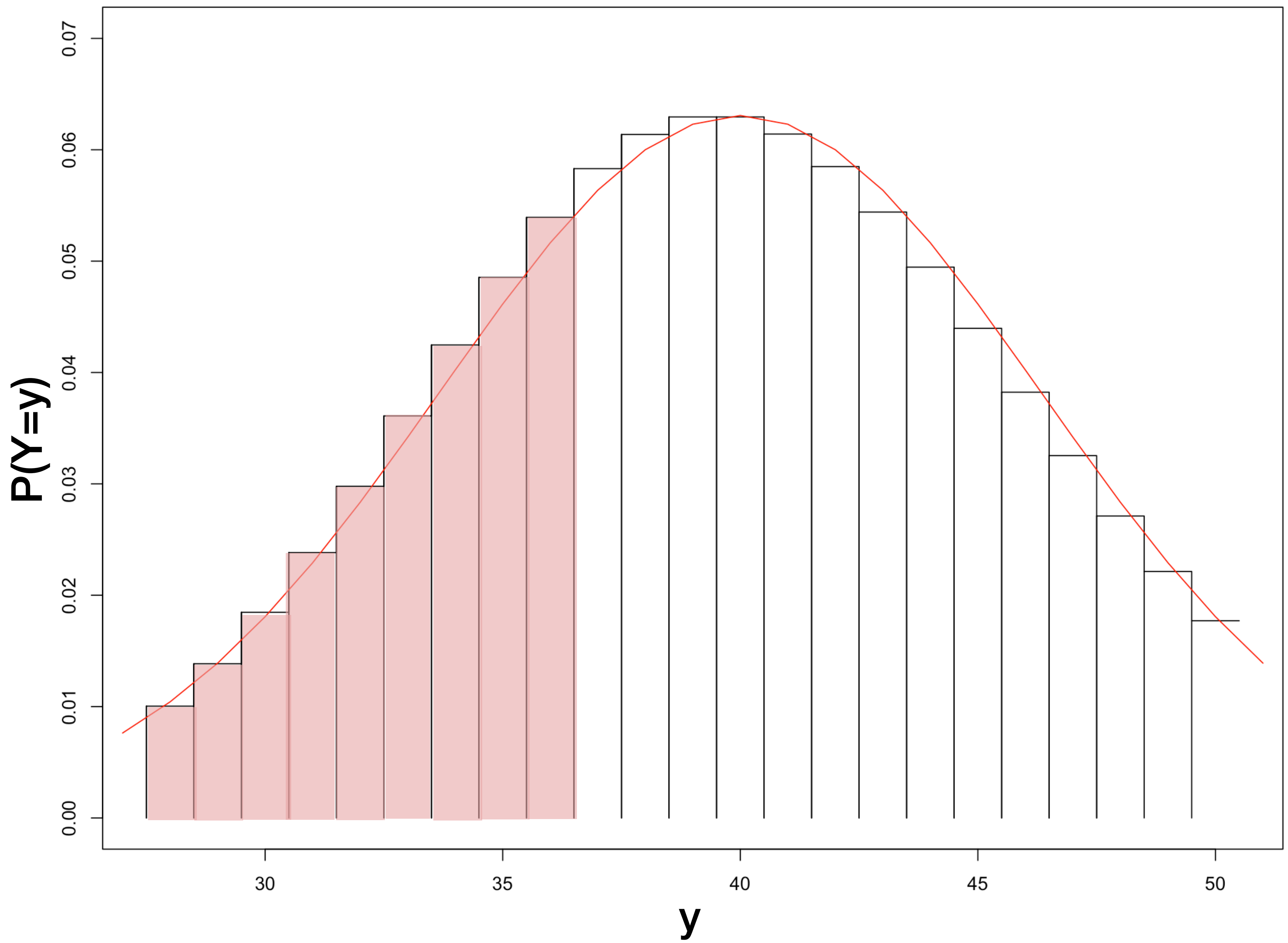
- Vi kan tilnærme denne summen ved å integrere under den røde kurven (sannsynlighetstettheten til normalfordeling som er sentrert rundt forventningen til Y og med samme standard avvik)

- For å få med hele søylen som representerer $P(Y = 36)$ må vi integrere opp til 36.5.



Sum av uavhengige Poissonfordelte stokastiske variabler

Innlevering 3, oppgave 1

- La $X_A \sim \text{Poisson}(\lambda_A)$ og $X_B \sim \text{Poisson}(\lambda_B)$ være to uavhengige poissonfordelte stokastiske variabler.

- Vi lar $Z = X_A + X_B$. Bestem punktsannsynligheten $f_Z(z) = P(Z = z)$ ved å bruke setningen om total sannsynlighet.

Sum av uavhengige Poissonfordelte stokastiske variabler

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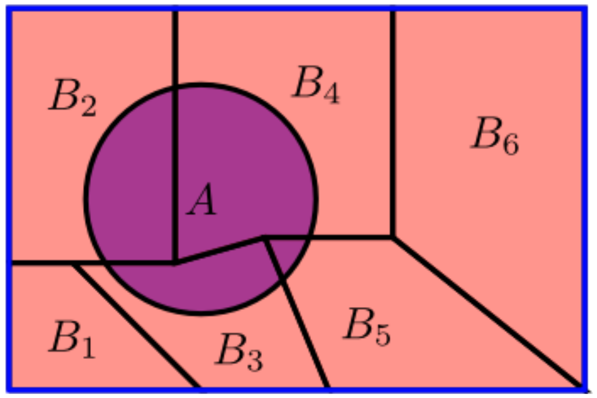
- Vi lar $Z = X_A + X_B$. Bestem punktsannsynligheten $f_Z(z) = P(Z = z)$ ved å bruke setningen om total sannsynlighet.

Teorem: Setningen om total sannsynlighet

La A være en hendelse i et utfallsrom S. La $B_1, B_2, \ldots, B_n$ være en partisjon av S, dvs $B_i \cap B_j = \emptyset$ for alle $i \neq j$ og $B_1 \cup B_2 \cup \ldots \cup B_n = S$. Da har vi at

$$P(A) = \sum_{i=1}^{n} P(A \cap B_i) = \sum_{i=1}^{n} P(A|B_i)P(B_i).$$



Fra blå bok s. 33

Noen rekker

$$\boxed{\sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k} = (a+b)^n}, \quad \sum_{k=0}^{\infty} \frac{a^k}{k!} = e^a \,, \quad \sum_{k=0}^{n} a^k = \frac{1-a^{n+1}}{1-a}.$$

$$\sum_{k=0}^{\infty} k a^k = \frac{a}{(1-a)^2} \,, \quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2} \,, \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$