TMA4250 Spatial Statistics
Assignment 2: Event Spatial Variables

IMF/NTNU/HO&TR

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Introduction

This assignment contains problems related to event spatial variables and Poisson random fields (RF) .The package R can be used for solving the problems and relevant functions may be found in the R library `spatial`.

Consider three real data point patterns in the R library `MASS`:

- biological cell data, available at `cells.dat`
- redwood tree data, available at `redwood.dat`
- pine tree data, available at `pines.dat`

Use the command `data<-ppinit("file.dat")` to load the data files.

Problem 1: Analysis of Point Patterns

Consider the three real data point patterns defined above.

a) Display each point pattern and discuss their appearance. Try to relate the point patterns to real processes in nature.

b) Compute the empirical L or J-function for each of the point patterns, the function `Kfn()` may be used.
Display the functions for each point pattern and discuss their appearance.

Specify the expression for the theoretical L and J-functions for a stationary Poisson RF. Display the empirical L or J-function for each of the point patterns with the corresponding theoretical function for a stationary Poisson RF, and discuss whether a stationary Poisson RF appear as a suitable model for each of the point patterns.

c) An empirical Monte Carlo test whether the stationary Poisson RF is a suitable model shall be performed. Consider each point pattern - under the hypothesis that the point pattern origin from a stationary Poisson RF, condition on the actual point count observed, and generate 100 realizations of the Poisson RF. For each realization compute the associated L or J-function and use the set of functions to empirically test whether the actual point pattern could origin from a Poisson RF.
Display the empirical 0.9-intervals for the functions jointly with the corresponding estimated functions for each point pattern. Discuss, for each point pattern, whether a Poisson RF is a suitable model.

Problem 2: Bayesian inversion in Poisson RF

Consider an area of size (300×300) m^2 containing a pine tree forest, and the actual locations of the pine trees are to be assessed. The pine tree locations are observed from a satellite by remote sensing, and due to partly cloudy weather the observation probability for individual trees vary across the area.

Discretize the area into a regular (30×30)-grid $\mathtt{L}$ with grid unit size 100 m^2, the true, but unknown, number of pine trees located in each grid unit is $\{k(\boldsymbol{x}); \boldsymbol{x} \in \mathtt{L}\}$. The probabilities for observing a pine tree in each grid unit is represented by $\{\alpha(\boldsymbol{x}); \boldsymbol{x} \in \mathtt{L}\}$ - and these probabilities varies across the area. The probabilities are listed in the file `obsprob.txt` together with the x and y coordinates of the centroids of the grid units.

The number of pine trees observed in each grid unit is given as $\{d(\boldsymbol{x}); \boldsymbol{x} \in \mathtt{L}\}$ - and they are listed in the file `obspines.txt`.

a) Display the observations and the observation probabilities. Assume that the observations given the true number of pine trees are spatially uncorrelated from one grid unit to another. Specify the expression for the corresponding likelihood model for the observations.

b) Assume apriori that the distribution of pine trees is according to a stationary Poisson RF with model parameter λ_k. Specify the expression for corresponding prior model for the discretized Poisson count model.

c) Estimate the intensity λ_k based on the observations with observation probabilities, and generate 10 realizations from this prior Poisson event-count model and the associated approximate Poisson event-location realizations. Display the approximate Poisson event-location realizations.

d) Develop the expression for the posterior discretized event-count model and justify that this posterior model is a Poisson RF model. Generate 10 realizations of the associated approximate event-location model, and display these realizations. Discuss the similarities and differences between the prior and posterior event-count and approximate event-location realizations.

Problem 3: Clustered event spatial variables

Consider the redwood tree data listed above.

a) Specify the expression for the Neuman-Scott (Mother-Child) cluster model for event spatial variables, and describe the model parameters.

Make an empirical fit of the model parameters to the redwood tree data, and justify your parameter values by a Monte Carlo test on the J or L-interaction function. Display graphically the test and the empirical estimates based on the redwood tree data. Discuss the results.

Problem 4: Repulsive event spatial variables

Consider the biological cell data listed above.

a) Specify the expression for the Strauss repulsion model for event spatial variables with a fixed event count, and describe the model parameters.

Make an empirical fit of the model parameters to the biological cell data, and justify your parameter values by a Monte Carlo test on the J or L-interaction function. Display graphically the test and the empirical estimates based on the biological cell data. Discuss the results.