Norwegian University
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Department of Mathematical Sciences

Contact during exam:
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EXAM IN COURSE 75563 SPATIAL STATISTICS

Friday May 26th 2000
Time: 0900 - 1300

Permitted aids:
- Statistiske tabeller og formler, Tapir
- Approved calculator
- Selfmade peep-sheet - A4 format

Lecturer:
- Prof. Henning Omre, Department of Mathematical Sciences; NTNU

Problem 1 CONTINUOUS FIELDS

Consider a continuous random field $\{R(x); x \in \mathbf{R}^1\}$. Note that the field has a one-dimensional reference. Assume:

$$E\{R(x)\} = 0$$
$$\text{Var}\{R(x)\} = 1$$
$$\text{Cov}\{R(x), R(x+h)\} = C(h) = \exp\{-h^2\}.$$

Define further the differential field:

$$\{R'(x) = \frac{dR(x)}{dx}; x \in \mathbf{R}^1\}$$

a) Which additional assumptions must be made in order for $\{R(x); x \in \mathbf{R}^1\}$ to be a Gaussian random field? Assume that $\{R(x); x \in \mathbf{R}^1\}$ is Gaussian, sketch graphically the bivariate probability densities (pdf) for:

$$[R(0), R(0.1)]$$
$$[R(0), R(1.0)]$$
$$[R(0), R(10.0)]$$

b) Specify the requirement for $\{R'(x); x \in \mathbf{R}^1\}$ to exist. Show that this is satisfied for $\{R(x); x \in \mathbf{R}^1\}$.

Demonstrate how

$$\text{Cov}\{R'(x), R(x+h)\}$$

can be computed, and find the expression.

What is the expression for

$$\text{Cov}\{R'(x), R'(x+h)\}$$

Sketch graphically the two covariance functions together with $C(h)$. Comment on the relation between $R(x)$ and $R'(x)$ in arbitrary $x \in \mathbf{R}^1$.

c) Assume that one has observed:

$$R'(0) = 0.5$$

Develope the best linear predictor for $\{R(x); x \in \mathbf{R}^1\}$ based on $R'(0) = 0.5$ under quadratic loss. Sketch graphically, and comment on, the results.

Problem 2 EVENT FIELDS

Consider a Poisson point field over $\mathbf{R}^2$ with intensity λ. Let $B_1 \subset \mathbf{R}^2$ and $B_2 \subset \mathbf{R}^2$ with $B_1 \cap B_2 = \emptyset$, be two disjunct domains, and $N(B_1)$ and $N(B_2)$ be the number of points in B_1 and B_2 respectively.

a) Specify the probability for observing exactly two points in B_1.
Assume that $N(B_1) = 2$.
Compute the expression for the conditional probability:

$$\text{Prob}\{N(B_2) = k | N(B_1) = 2\}$$

b) Consider an arbitrary location $x_0 \in \mathbf{R}^2$, and define:

$R_{(1)}$ - distance from x_0 to the closest point in the Poisson field
$R_{(2)}$ - distance from x_0 to the second-closest point in the Poisson field.

Develope the probability density (pdf) for the variable $R_{(2)}$.
Develope the probability density (pdf) for the bivariate variable $(R_{(1)}, R_{(2)})$.
Demonstrate that the two results are consistent.

Problem 3 MOSAIC FIELDS

Consider the random variable $\{L_x; x \in \mathcal{L}_D\}$ with $\mathcal{L}_D$ being a grid over the domain $D \subset \mathbf{R}^2$. Let the sample space for L_x be discrete, i.e. $L_x \in \{1, ..., K\}$; for all $x \in \mathcal{L}_D$, and let it be positive probability for all outcomes.

a) Assume that $\{L_x; x \in \mathcal{L}_D\}$ is a Markov field with 3×3-neighborhood.

What does the Markov assumption entail?

Specify and interpret the expression for the corresponding Gibbs field.

What is the major message in the Hammersley-Clifford theorem?