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EKSAMEN I FAG SIF5064 ROMLIG STATISTIKK

Tirsdag 14. mai 2002
Tid: 0900-1300

Hjelpemidler:
- Statistiske tabeller og formler, Tapir
- Godkjent lommekalkulator
- Egetprodusert gult titte-ark - A4-format

Faglærer:
- Prof. Henning Omre, Institutt for matematiske fag; NTNU

Sensuren faller i uke 23.

Problem 1 CONTINUOUS FIELDS

Consider a continuous random field $\{R(x); x \in \mathcal{D} \subset \mathbb{R}^2\}$ with

$$E\{R(x)\} = \sum_{i=0}^{3} a_i g_i(x) = \mu(x); \text{ all } x \in \mathcal{D}$$

where $g(x) = (g_0(x), \ldots, g_3(x))$ are known functions, with $g_0(x) = 1$, and $a = (a_0, \ldots, a_3)$ are unknown constants.

Further, the covariance function is:

$$\text{Cov}\{R(x'), R(x'')\} = C(\tau) \text{ ; all } x', x'' \in \mathcal{D}$$

where $\tau = |x'' - x'|$. Consider $C(\tau)$ to be unknown.

a) Which constraints must be set on the covariance function in order for the model to be valid? State this in mathematical form.

Assume now that the field is observed in 50 arbitrary locations:

$$R^0 : \{R(x_1), \ldots, R(x_{50})\}$$

b) Describe how you would estimate the covariance function, $C(\tau)$, based on these observations. Which problems appear? Mention also shortly alternative estimation procedures.

Assume in the following that the covariance function, $C(\tau)$, is determined and can be considered known.

c) A prediction of the field in an arbitrary, non-observed location $x_0 \in \mathcal{D}$ shall be made. Hence a prediction of $R(x_0)$, based on the observations R^0 shall be made.

Develope the best, linear, unbiased predictor under quadratic loss (BLUP). It is sufficient to define the minimazation system to be solved.

d) Assume now that only two observations are available:

$$R^{00} : \{R(x_1), R(x_2)\}$$

Which problems do then occur in the prediction in question c)?

If the coefficients in the expectation function $E\{R(x)\}$ are considered to be random, $A = (A_0, \ldots, A_3)$, with known moments $E\{A\} = \mu_A$ and $\text{Cov}\{A\} = \sum_A$, then the problem can be solved.

Show how.

Problem 2 EVENT FIELDS
Consider a stationary point field over $\mathbb{R}^2$ with intensity λ.

Assume that the field is observed by counting the number of points in seven arbitrarily located, non-overlapping, circular domains with varying radius:

radius [cm]	no. of points
3	5
6	24
9	47
12	82
15	123
18	183
21	248

a) Specify an estimator for the intensity λ. Determine the estimate.

b) How can the hypothesis conserning a Poisson point field be evaluated.

c) Assume that it is a Poisson point field. Specify the maximum likelihood estimator for the intensity λ.

Outline also an alternative estimator which draws on the Poisson point field assumption.

Problem 3 MOSAIC FIELD

Consider a random field $L : \{L_x; x \in \mathcal{L}_\mathcal{D}\}$ where $\mathcal{L}_\mathcal{D}$ is a grid over the domain $\mathcal{D} \subset \mathbb{R}^2$. Let the sample space be $L_x \in \{-1, 1\}$ for all $x \in \mathcal{L}_\mathcal{D}$. Assume that the joint distribution can be written as:

$$\text{Prob}\{L = l\} = \text{const} \times \exp\{\beta \sum_{x \sim y} I(l_x = l_y)\}$$

with $I(A)$ being 1 if A is true and 0 else, and $x \sim y$ denoting all closest neighbors, ie all horizontal and vertical neighbors. Hence the field is an Ising-field.

Assume that the model parameter β is unknown.

Let $L = l^0$ be an observed outcome of the field.

a) An estimate for β shall be found.

Spesify the likelihood function for the model above, and explain why it is difficult to use in the estimation of β.

Define the pseudo-likelihood function for the model above, and develope the maximum pseudolikelihood estimator for β. It is sufficient to determine the system of equations to be solved.

Ignore the border effects in all definitions and developments above.