**Norges teknisk–
naturvitenskapelige universitet
Institutt for matematiske fag**

English

Faglig kontakt under eksamen:
Jo Eidsvik Tel: 90 12 74 72

EXAM IN COURSE TMA4250 SPATIAL STATISTICS

Monday 22 May 2006
9.00-13.00

Permitted aids: Calculator HP30S
Statistiske tabeller og formler, Tapir forlag.
Self made peep-sheet - A4 format

The results will be announced in week 24

Problem 1 CONTINUOUS FIELD

Consider a stationary Gaussian random field (RF) $\{R(x); x \in \mathcal{R}^1\}$ in 1D, with model parameters:

$$\begin{aligned} \mathrm{E}\{R(x)\} &= 0 \\ \mathrm{Cov}\{R(x'), R(x'')\} &= \begin{cases} 1 & \text{for } x' = x'' \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

a) Assume that the following observations are made : $R(-1) = r_{-1}$ and $R(1) = r_1$. Consider $R(0)$ and develop the kriging predictor and the kriging variance.

For a given $d \geq 0$, define the related RF:

$$\left\{ S_d(x) = \int_{x-d/2}^{x+d/2} R(y)dy; x \in \mathcal{R}^1 \right\}$$

b) Explain why $\{S_d(x); x \in \mathcal{R}^1\}$ is a Gaussian RF. Develop the expression for the associated model parameters.

c) Assume that the following observations are made : $S_1(0) = s_{10}$ and $S_3(0) = s_{30}$. Consider $S_2(0)$ and develop the kriging (actually co-kriging) predictor and kriging variance.

Problem 2 EVENT FIELD

Consider a homogeneous Poisson point random field (RF) with intensity λ on $\mathcal{R}^2$. Let $x_0 \in \mathcal{R}^2$ be an arbitrary location in $\mathcal{R}^2$, and define $R_{(i)}$ to be the distance to the i'th closest point in the RF from x_0.

a) Specify the definition of a Poisson point RF on $\mathcal{R}^2$.

If the Poisson point RF is constrained to be in a finite domain $\mathcal{D} \subset \mathcal{R}^2$ an alternative definition is available. Specify this alternative definition.

b) Show that

$$f(r_{(1)}, \ldots, r_{(k)}) = \begin{cases} [2\lambda\pi]^k \prod_{i=1}^{k} r_{(i)} \exp\{-\lambda\pi r_{(k)}^2\}; & 0 < r_{(1)} < \cdots < r_{(k)} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

is the correct joint pdf for $[R_{(1)}, \ldots, R_{(k)}]$.

c) Assume that only the closest and the third closest points are located, ie only $r_{(1)}$ and $r_{(3)}$ are known.

Develop a maximum likelihood estimator for λ based on $r_{(1)}$ and $r_{(3)}$.

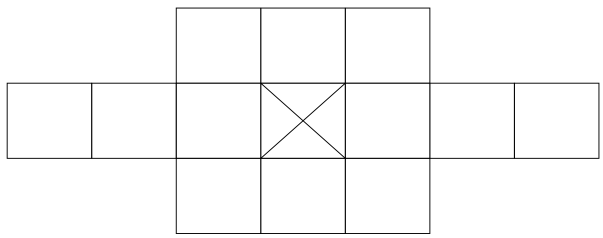
Problem 3 MOSAIC FIELD

Consider a Markov random field (RF) $L : \{L_x; x \in \mathcal{L}_\mathcal{D}\}$ where $\mathcal{L}_\mathcal{D}$ is a regular grid over the domain $\mathcal{D} \subset \mathcal{R}^2$ and $L_x \in \Omega = \{l^1, \ldots, l^k\}$. This RF may be specified by the conditional probabilities:

$$\text{Prob}\{L_s = l_s | L_t = l_t; t \in \partial_s\};\ l_s \in \Omega\ ;\ \forall s \in \mathcal{L}_\mathcal{D}$$

where $\partial : \{\partial_s; s \in \mathcal{L}_\mathcal{D}\}$ is a neighbourhood system.

The neighbourhood system is defined by each pixel (sufficiently far from the borders) having twelve neighbours as shown in the figure:



a) Specify the Gibbs RF model which is associated with the Markov RF model.

Specify in particular the geometry of the largest cliques associated with the neighbourhoods presented above.