Exercise 2-Solutions
TMA4255
Applied Statistics

February 7, 2017

1 Problem 1

The problem can be solved using MINITAB, after have saved the data in the first column, running the following commands:

Commands: *Stat − > Basic statistics − > Display Descriptive statistics*

2 Problem 2

From the Central Limit Theorem we know that:

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$$

so we can compute

$$P\left(\sum_{i=1}^{36} x_i \geq 1458\right) =$$

$$P(36\bar{X} \geq 1458) = P(\bar{X} \geq 40.5) = P\left(\frac{\bar{X} - 40}{2/\sqrt{36}} \geq \frac{40.5 - 40}{2/\sqrt{36}}\right) =$$

$$P(Z \geq 1.5) = 1 - P(Z < 1.5) = 0.06681$$

3 Problem 3

$$P(\mu_{\bar{X}} - 1.9\sigma_{\bar{X}} \leq \bar{X} \leq \mu_{\bar{X}} - 0.4\sigma_{\bar{X}}) = P(\bar{X} \leq \mu_{\bar{X}} - 0.4\sigma_{\bar{X}}) - P(\bar{X} < \mu_{\bar{X}} - 1.9\sigma_{\bar{X}})$$

Now using the Central Limit Theorem similarly to the previous problem we have $Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} \sim N(0, 1)$. Hence

$$P(\bar{X} \leq \mu_{\bar{X}} - 0.4\sigma_{\bar{X}}) - P(\bar{X} < \mu_{\bar{X}} - 1.9\sigma_{\bar{X}}) = P\left(\frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} \leq -0.4\right) - P\left(\frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} < -1.9\right)$$

$$P(Z \leq -0.4) - P(z < -1.9) = 0.3159$$

4 Problem 4

$$P(19.9 \leq \bar{X} \leq 20.1) = P(\bar{X} \leq 20.1) - P(\bar{X} < 19.9) =$$

$$P\left(\frac{\bar{X} - 20}{3/\sqrt{n}} \leq \frac{20.1 - 20}{3/\sqrt{n}}\right) - P\left(\frac{\bar{X} - 20}{3/\sqrt{n}} < \frac{19.9 - 20}{3/\sqrt{n}}\right) =$$

$$P(Z \leq 0.03\sqrt{n}) - P(Z \leq -0.03\sqrt{n}) =$$

$$2P(Z \leq 0.03\sqrt{n}) - 1$$

Since we know that $P(19.9 \leq \bar{X} \leq 20.1) = 0.95$, we have

$$2P(Z \leq 0.03\sqrt{n}) - 1 = 0.95 \Rightarrow P(Z \leq 0.03\sqrt{n}) = 0.975$$

$$\Rightarrow 0.03\sqrt{n} = 1.959964 \Rightarrow n = 4268$$

5 Problem 5

Using theorem 8.8 of the book we have that $F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} = \frac{S_1^2\sigma_2^2}{S_2^2\sigma_1^2}$ has an F-distribution with $\nu_1 = n_1 - 1$ and $\nu_2 = n_2 - 1$ degrees of freedom.

$$P\left(\frac{S_1^2}{S_2^2} > 1.26\right) = P\left(\frac{S_1^2\sigma_2^2}{S_2^2\sigma_1^2} > 1.26\frac{\sigma_2^2}{\sigma_1^2}\right) = P(F > 1.89) = 0.05$$